ATTACHMENT C
Determining Provisionally Winning Bids and Current Price Estimates

The licenses subject to package bidding in a Hierarchical Package Bidding auction are grouped into non-overlapping, successively larger packages at each level. For purposes of this explanation, we are using an HPB auction as proposed in this Public Notice − with two levels, with 12 individual REAG licenses in the first level and several packages of different sizes in the second level, as shown in Table 1 below. After each round, all considered bids and minimum opening bids are used to determine the largest bid on each license and on each allowed package, using the random tie-breaking rule.[1] A simple procedure is used to calculate the provisionally winning set of bids and a "current price estimate" ("CPE") for each individual license.[2] This appendix describes the winner determination and pricing rules, first in terms of an example and then in more general terms.

Table 1. Two-Level Structure

Level 2: Packages	50 States								Atlantic		Pacific	
Level 1: REAG licenses	R1	R2	R3	R4	R5	R6	R7	R8	R10	R12	R9	R11

Determining Provisionally Winning Bids

After each round of bidding, the FCC Auction System determines which combination of bids together provides the greatest aggregate gross amount and are therefore provisionally winning. In order to determine the best combination of bids, the system takes the highest bid on each individual license. It then compares the sum of the high bids on individual licenses in a package with the highest package bid to determine provisionally winning bid(s).

Example 1: Suppose that the highest bids on each license and package are as shown in Table 2 below. The sum of the highest bids on REAG licenses R1-R8, 80, exceeds the highest bid on the 50-state package, 70; the sum of the highest bids on REAG licenses R10 and R12, 20, exceeds the highest bid on the Atlantic package, 15; and the sum of the highest bids on REAG licenses R9 and R11, 20, exceeds the highest bid on the Pacific package. Thus, the provisionally winning bids would be the bids on the individual REAG licenses.

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[1] The set of considered bids in a round includes new bids for that given round and each bidder's highest bid on each license or package, regardless of the round in which it was placed. For a description of how tied bids are broken, see Section IV.C.6. "Provisionally Winning Bids."

[2] The hierarchical structure of HPB was suggested by Rothkopf, Pekeč, and Harstad, "Computationally Manageable Combinatorial Auctions," *Management Science*, 44(8), August 1998, 1131-1147. The pricing mechanism for HPB was proposed by Goeree and Holt (2007) "A Simple Combinatorial Auction," Working paper, May 2007.

Table 2. Example 1 with High Bids in Parentheses

Level 2: Packages	50 States (70)								Atlantic (15)		Pacific (15)	
Level 1: REAGs	R1 (10)	R2 (10)	R3 (10)	R4 (10)	R5 (10)	R6 (10)	R7 (10)	R8 (10)	R10 (10)	R12 (10)	R9 (10)	R11 (10)

Example 2: Next, suppose that the highest bid on the 50-state package changes to 120, as shown below in Table 3, which exceeds the sum of the high bids for the 8 individual REAGs included in the package. In this case, the provisionally winning bids would be the package bid for the 50 states and the bids for the 4 individual REAG licenses in the Atlantic and Pacific packages.

Table 3. Example 2 with High Bids in Parentheses

Level 2: Packages	50 States (120)								Atlantic (15)		Pacific (15)	
Level 1: REAGs	R1 (10)	R2 (10)	R3 (10)	R4 (10)	R5 (10)	R6 (10)	R7 (10)	R8 (10)	R10 (10)	R12 (10)	R9 (10)	R11 (10)

Determining Current Price Estimates (CPEs) for Licenses

CPEs for licenses are determined in such a way as to indicate to bidders how high they must bid to compete with the provisionally winning bid on the package containing those licenses in the event the package bid is the provisionally winning bid. If the bids on the individual REAG licenses are all provisionally winning, then the CPEs are simply equal to those bids, as is the case in a standard simultaneous multiple round (SMR) auction. But if the bid amount for a package is greater than the sum of the bid amounts for the individual licenses it contains, then the highest bids received so far for the licenses are scaled up to the package bid amount by adding "shares."[3] These shares are proportional to the bidding units associated with each license relative to the total number of bidding units in the package.

Suppose that all REAG licenses have equal bidding units, and that the bid amounts are as shown as in Example 2 above. The high bid on the 50 States package is 120, which exceeds the sum of the bids for the REAG licenses in the package. Therefore, the individual bids for licenses R1-R8 are raised from 10, to $10 + (1/8)[120 - 80] = 15$. In other words, the "shortfall" of 40 is divided among the 8 licenses in the package in proportion to their bidding units (assumed to be equal). In this example, a "package share" of 5 is added to the high bids on the REAG licenses in the 50-state package, but there is no package share for the licenses in the other packages.

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[3] If no bids have been placed on an individual license, the share will be added to the minimum opening bid amount.

Minimum acceptable bids for licenses in the next round of the auction are calculated by adding a percentage to the license CPE.[4] For packages, minimum acceptable bids are the sums of the minimum acceptable bids for the licenses in the package. As a result of constructing CPEs in this way, the minimum acceptable bids for the next round will indicate to bidders what they must bid to unseat the provisionally winning bids from the last round.

Technical Description

To Determine Provisionally Winning Bids: Suppose a package r is divided into individual licenses, $i = 1,\dots,I_r$ where I_r is the number of individual licenses in package r, each with α_{ir} bidding units. Bidders can submit bids on individual licenses and package bids for the entire package. Let $b_{ir}^{\max}$ denote the highest bid for license i in package r and $b_r^{\max}$ the highest package bid for package r. In this case, the revenue-maximizing allocation can be found recursively (Rothkopf, Pekeč, and Harstad, 1998):

1. Pick the larger of $b_r^{\max}$ and $\Sigma^{Ir}_{i=1} b_{ir}^{\max}$, which determines Rev_r, the revenue for package r.

2. If $Rev_r = b_r^{\max}$, the highest package bid for package r is provisionally winning and the bids on individual licenses in that package are losing, otherwise the highest bids on individual licenses in package r are provisionally winning and the package bid for package r is losing.

To Determine CPEs for Licenses: A second procedure is used to calculate current price estimates (CPEs) for licenses (Goeree and Holt, 2007).[5] The CPE p_{ir} of license i in package r is the maximum bid for the license plus a "package share" (if the sum of the high bids for individual licenses in the package falls short of Rev_r). The shares are determined by the relative sizes of the licenses in terms of bidding units. Let $\alpha_r = \Sigma^{Ir}_{i=1}\alpha_{ir}$ denote the total number of bidding units in package r. With this notation, the pricing formula is:

$$p_{ir} = b_{ir}^{\max} + \frac{\alpha_{ir}}{\alpha_r}(Rev_r - \sum_{i'=1}^{I_r} b_{i'r}^{\max})$$

where the difference in parentheses is non-negative by construction. Thus,

- $p_{ir} = b_{ir}^{\max}$ if an individual license bid wins.

- $p_{ir} = b_{ir}^{\max}$ + package share if a package bid wins.

Current price estimates based on this procedure address the "threshold problem" in the sense that they provide benchmarks for bids on individual licenses that will, when incremented, allow the individual license bids to displace a provisionally winning bid on the package containing those licenses.

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[4] In a standard FCC SMR auction (without package bidding), the FCC Auction System adds a percentage to the current provisionally winning bid amount for each license to determine minimum acceptable bids for the next round. This percentage may be determined by the activity-based formula, described in Attachment B and in Section IV.C.5. "Bid Amounts."

[5] The computed prices determine minimum acceptable bids for licenses by adding a percentage. The minimum acceptable bid for a package is equal to the sum of the minimum acceptable bids for the licenses it contains.

The procedures for determining provisionally winning bids and CPEs for licenses, set forth here for two levels, can be generalized for any number of levels. With a single level these procedures reduce to the SMR pricing and allocation rules in which the provisionally winning bid for a license is the highest bid received for that license. With multiple levels, the algorithm for determining provisionally winning bids follows a "bottom-up" approach and checks at every level, h, whether it is better to provisionally assign a level-h package to the high bidder for that package or to provisionally assign the smaller sub-packages that it contains. The rule for determining CPEs follows from this recursive comparison of level-h and level-$(h+1)$ packages: if the larger packages provide higher revenue, then the prices of the smaller packages (at the less aggregated level) are incremented in proportion to bidding units to reflect this shortfall. Once the algorithm finishes, the CPEs of the individual licenses add up to the sum of the associated provisionally winning bids.