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41 Enhancing Spectrum's Value Through Market-informed Congestion Etiquettes

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Abstract

This paper evaluates the ability of different wireless spectrum congestion etiquettes to promote the efficient use of wireless spectrum in the presence of licensed and unlicensed operations. Under the examined environment, theory predicts that society leaves half of the value it can receive from spectrum "on the table." One new approach utilizes various types of user information to address the inefficient use problem. The superiority of this new class of etiquette is established both in theory and by experimental results. Assuming a close similarity between the naturally occurring environment and the experimental one, analysis reveals that the average efficiency of the existing etiquette employed in most unlicensed equipment is 42%. In comparison, experimental analysis reveals that the average efficiency of one market-informed etiquette - the Informed Greedy Algorithm - is 70%. These results form the factual basis for generating an entirely new type of spectrum allocation wherein a given band of spectrum is treated as a common pool resource in the absence of excessive spectrum congestion, but is treated as an excludable private good in the presence of such congestion.

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1 Introduction

The development and growing acceptance of bandwidth-intensive applications (e.g., video and user generated content) has made it more difficult for wireless network operators to manage their transmission capacity and minimize congestion. While innovation and network investment may alleviate some of the problem, these activities take time to complete and, thus, do little to address existing congestion problems. Moreover, there is the issue of whether increases in supply can ever keep up with increases in demand. Congestion concerns appear highest under operations where spectrum is treated as a common pool resource. Like other forms of congestion (e.g., highway and airport congestion), wireless spectrum congestion imposes an important and real cost on society. This cost can be reduced significantly if the market for radio spectrum worked in a manner that better assigned existing spectrum capacity to those users that value it the most.

This study employs economic methods to evaluate the efficiency with which spectrum is assigned to users when spectrum is allocated to both licensed and unlicensed operations. The study places particular emphasis on unlicensed operations. The source of the economic problem related to unlicensed operations is straightforward. Markets work best when the choices economic actors make reflect the cost their decisions impose on society.[2] Economic agents typically incur the cost of their decisions by paying an appropriately informed market price. Due to the free entry, open access condition inherent in unlicensed use, there is no assurance that consumers will take into account the negative effect of their spectrum consumption decisions on the value other consumers place on using the same spectrum. Thus, rational users may "over-consume" freely available spectrum and "under-consume" non-freely available spectrum compared to the levels that would promote both consumer and society's interests. This welfare reducing outcome is referred to as the "Tragedy of the Commons."

Based on two simple examples that contain many of the economic features that exist in the naturally-occurring world, economic theory indicates that society leaves half of the value it can receive from spectrum "on the table." Fortunately, theory suggests ways in

[2] Simultaneous use of radio waves at the same point in time and space by independent users makes it difficult for receivers to differentiate between signals and, thus, differentiate between wanted and unwanted information.

which the efficiency of the spectrum market can be improved. One approach involves designing better etiquettes that determine who gets assigned spectrum and how much. One examined etiquette employs market clearing prices during periods of excessive spectrum congestion to address the congestion problem. The superiority of a new class of etiquette - herein referred to as "market-informed" etiquettes - is established both in theory and by experimental results. Indeed, both theory and experimental evidence indicate that society can experience a substantial increase in its welfare if it employs one or more members of this new class of etiquette to address wireless spectrum congestion. Importantly, analysis also indicates that, despite paying a fee to access spectrum under conditions of excess congestion, consumers are in general much better off under these new market-informed etiquettes than under the standard, existing congestion etiquette. These results form the factual basis for generating an entirely new type of spectrum allocation wherein a given band of spectrum is treated as a common pool resource in the absence of excessive spectrum congestion, but is treated as an excludable private good in the presence of such congestion. Results of this analysis have implications for a variety policy issues. For example, by reducing concerns that the unassigned frequencies between broadcast TV channels – so-called "white spaces" – will be over-used, the value society receives from the FCC's proposal to allow unlicensed devices in these channels can be greater than anticipated.

2 Formal Analysis of the Tragedy of the Commons

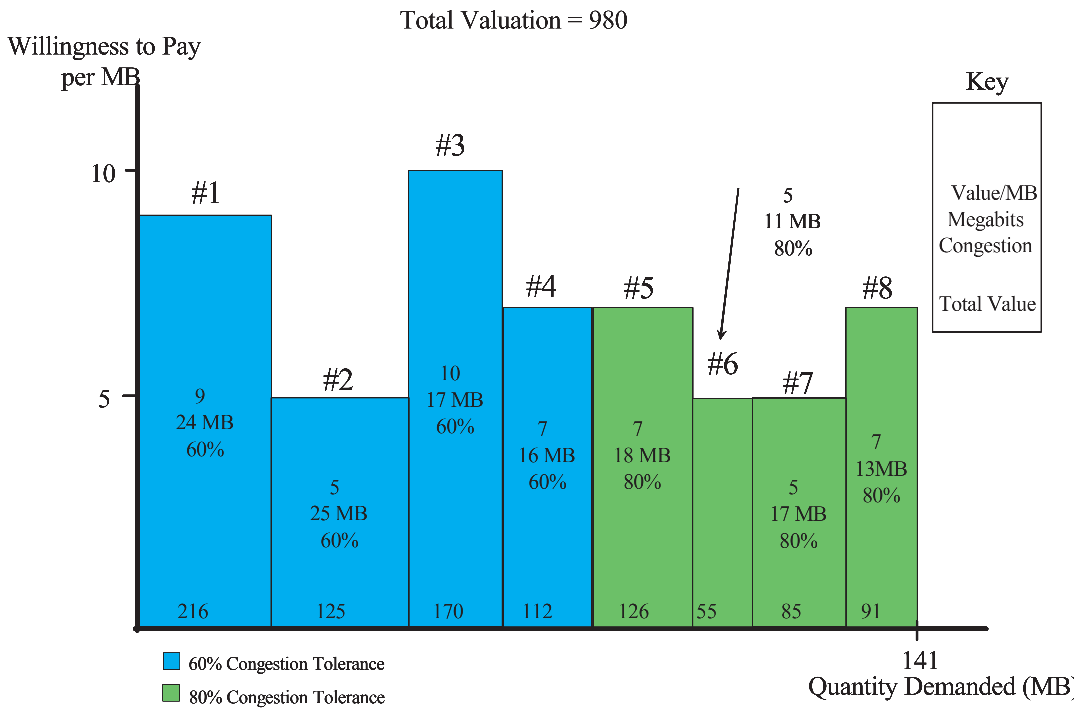
The Tragedy of the Commons refers to a situation in which myopic, self-interested behavior leads to the excessive use of a common pool resource. Because of this excessive use, the welfare of individual users is not maximized and, further, society fails to obtain the most efficient use of its scarce resources. Importantly, users of a common pool resource (e.g., spectrum designated to unlicensed use) can avoid the Tragedy of the Commons problem by coordinating their use. While such coordination has been successful in other instances, the radio spectrum market has several characteristics that make user coordination particularly difficult.

One market characteristic is the wide variation in uses to which entities employ spectrum. Some users employ spectrum to download streaming video, while other users employ spectrum to text message. The level of satisfaction a user obtains from employing spectrum depends importantly on the amount of unused capacity available to satisfy his or her desired application. Equally important, the level of available capacity needed to provide satisfactory streaming video service is different (i.e., substantially greater) than the level needed to provide satisfactory text messaging service. Such differences in the amount of spectrum required by different applications complicate the ability of users to coordinate their uses. In particular, to avoid the Tragedy of the Commons problem, users must not only recognize that their spectrum use negatively affects other users, but that the extent to which it affects other users depends on the specific demands of other users. The economic cost of adding one additional user who wishes to download streaming video to a wireless network may be zero if all current users are employing spectrum for text messaging, but may be very high if existing users, because of their applications, are relatively spectrum congestion intolerant.

2.1 A Simplified Model

A simple example can be used to describe the unique and complicated nature of the Tragedy of the Commons problem involving spectrum usage. Assume that there are eight different spectrum users, each defined by a set of characteristics. The users vary in the minimum amount of bandwidth that each needs in order to satisfy their service needs, as well as in the value they place on having their service needs fully satisfied. Because of differences in service needs, users also vary in the extent to which they can tolerate spectrum congestion. For example, a user requiring a very high quality of service (e.g. video streaming or other applications that require a high and reliable data transmission rate) might find quality unacceptable whenever aggregate demand exceeds 60% of system capacity. Users that can tolerate a lower quality of service, however, would find quality acceptable when aggregate demand exceeds a higher percentage of system capacity (e.g. 80%). In what follows we measure a user's demand for service quality by his or her "congestion limit," expressed as the maximum amount of spectrum congestion the user can tolerate before the value he or she places on employing spectrum falls from some desired value to zero.[3] Figure 1 presents data for a hypothetical set of spectrum users, including the value that each user places on spectrum (expressed on a per megabit basis), the minimum amount of spectrum required by the user, and each user's quality of service requirement as measured by his/her congestion tolerance limit.

Figure 1: **Spectrum User Characteristics in Example 1**



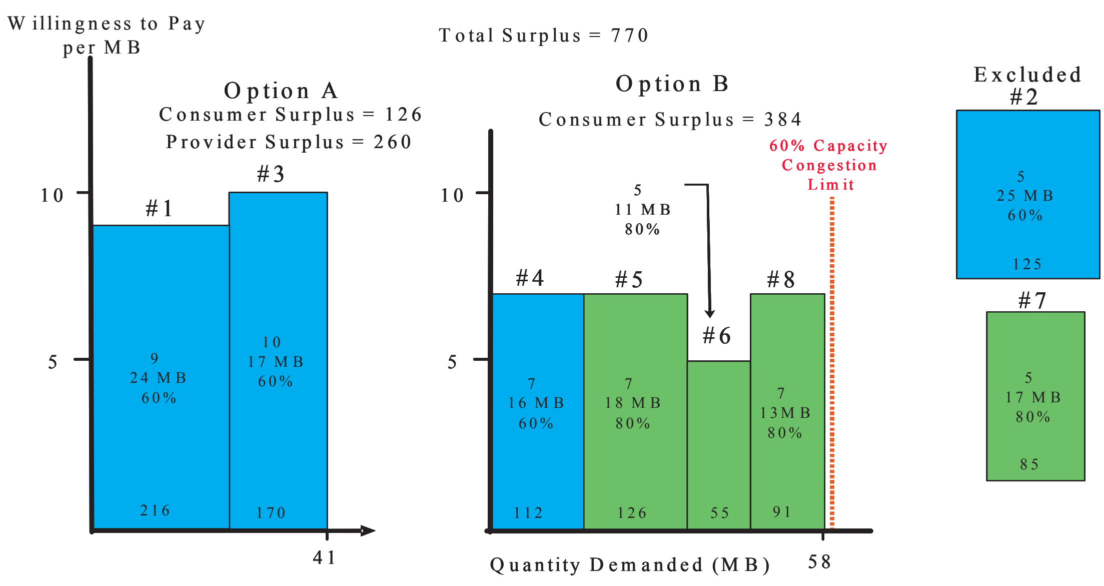
Spectrum users increasingly face a choice between using either dedicated spectrum provided through a subscription service or using shared spectrum allocated to unlicensed

[3] A user's need for quality is in fact a demand for a number of data transmission characteristics including file transfer rate, latency tolerance, and allowable level of jitter. See in Appendix A: Figures and Charts. For purposes of the experiments, we have assumed that the relationship between service quality and spectrum congestion for congestible service (defined later as Option B) follows an "Ethernet curve." At low levels of congestion users can obtain bit rates close to the maximum the air link can support. As system demand approaches 80% of total system capacity, the bit rate drops precipitously. For example, when demand is equal to 90% of capacity, bit rates are approximately 50% of the maximum possible. See Figure A2. Ethernet Packet Loss Function in Appendix A: Figures and Charts. The Ethernet curve can be well approximated by a function f of $z = X/K$, where K is maximum capacity, $f(0) = 1$, $f(1) = 0$ and $f'(z) < 0$ for all z such that $0 < z < 1$. In the theoretical analysis of this paper, and in the experiments based on that theory, we approximate this function by a simple step function in which user valuations are constant until demand reaches a critical percentage of available capacity. This "congestion limit" is allowed to vary by user type.

devices.[4] Our model, therefore, assumes that each user in our example has the option to choose between a reliable and non-congestible transmission system that requires a fixed payment of 130 units (Option A), and a transmission system that is "free," but where the value to each user depends on the congestion level of the free service's transmission system (Option B). The capacity of the free service is assumed to be 100 units. The identification of the efficient assignment of spectrum involves considering the amount of spectrum demanded by the users, the value users place on this spectrum, and the minimum quality of service that each user needs to obtain that value.

The unique efficient assignment of spectrum in this example involves assigning spectrum to users as shown in Figure 2, where total surplus is defined as $\sum_{i \in A} v_i q_i + \sum_{i \in B} v_i q_i$, consumer surplus is defined as $\sum_{i \in A} (v_i q_i - P_A) + \sum_{i \in B} (v_i q_i - P_B)$, q_i represents user i's demand, v_i represents value per unit of demand for user i, and P_A and P_B represent the prices for options A and B respectively.[5]

Figure 2: Efficient Assignment of Spectrum in Example 1



[4] For a discussion of spectrum sharing in the context of the larger debate on spectrum management reform, see Peha, Jon, "Emerging Technology and Spectrum Policy Reform," *International Telecommunications Union (ITU) Workshop for Spectrum Management, ITU Headquarters, Geneva,* January, 2007.

[5] By default, the price of Option B is equal to zero. Under some etiquettes, to be discussed later, a positive price might be assigned. In the following section, a per unit price, P_B , may be assigned to users of Option B. In the above notation, producer surplus is defined as $P_A\, n_A + P_B\, n_B$, where n_A and n_B represent the number of subscribers choosing options A and B respectively.

Notice that in the efficient assignment, users 2 and 7 are excluded from spectrum available via Option A or Option B. In particular, under Option A, both users 2 and 7 would receive negative surplus, because the value they place on service is less than the subscription fee of 130. Moreover, including either user 2 or user 7 under the free service (Option B) would require displacing at least one of the existing users of that service, and one can verify that any such change would result in a reduction in total surplus.

2.2 Tragedy of the Commons Problem: Nash Prediction

Suppose users simultaneously and independently choose whether to select the subscription service (Option A), the free service (Option B), or neither service. A Nash equilibrium is a particular joint selection such that no individual user could increase his or her payoff by selecting an alternative strategy assuming that all other users maintain their equilibrium strategy choices. We can now see immediately that the efficient assignment in Example 1 is not a Nash equilibrium. For example, suppose that user 7 contemplates joining the free service. Such a change would increase the total demand for this service from 58 to 75 units, but since user 7 expects to receive acceptable quality of service as long as total demand is less than 80 units, this alternative selection would increase user 7's surplus from zero to 85 valuation units.[6] Under Nash equilibrium assumptions, user 7 would therefore choose this alternative. With a total demand of 75 for Option B, however, user 4 no longer receives a satisfactory quality of service, and under our assumptions, her entire surplus of 112 valuation units is assumed to be forfeited.

While the efficient assignment cannot be sustained as a Nash equilibrium, there are alternative joint user selections that satisfy the Nash equilibrium conditions. The magnitude of the Tragedy of the Commons problem in Example 1 is measured by the difference between the total surplus under the efficient assignment and total surplus under the Nash equilibrium condition. The value of this measure, as predicted by Nash Equilibrium Theory, depends on the joint user selection. One common element of each Nash assignment is that both users 1 and 3 are assigned to the subscription service

[6] User 7 has a congestion tolerance of 80%. Given that the free service's capacity is 100 Mb, user 7 would obtain value from selecting Option B

(Option A). To see this, suppose that users 1 and 3 select Option A, users 5, 6, 7, and 8 select Option B, and that users 2 and 4 choose neither option. These selections satisfy the Nash equilibrium conditions. If either user 1 or user 3 attempts to switch to the free service in order to avoid paying the subscription fee, demand for Option B would rise above either of their congestion limits and they would forfeit the value received by selecting Option A. Each of the users choosing Option B would clearly see a reduction in surplus by switching to Option A since they would be required to pay the subscription fee of 130, under which they receive negative surplus. Finally, users 2 and 4 receive zero surplus by choosing neither option, but would do no better by choosing either Option A or Option B.

Total surplus in this equilibrium selection is equal to 743 valuation units. In fact, this is the highest total surplus obtainable in any of the Nash equilibria, although it is lower than the surplus of 770 obtained under the efficient assignment. Thus, if this equilibrium was obtained, the social cost of the Tragedy of the Commons problem would be very small. However, as noted earlier, there are many joint user selections that satisfy the Nash equilibrium conditions, many of which generate much lower levels of total surplus.

While there a large number of outcomes predicted under the assumption that users follow Nash equilibrium behavior, there are strong theoretical grounds for assuming that the outcomes that generate the greatest amount of social loss are the ones that are the most likely to occur. To see this, note that users who select Option B are guaranteed to receive at least a zero payoff, and there is a possibility that they could receive a positive payoff if other users deviate (contrary to Nash assumptions) from the assumed equilibrium. Those who select "neither option" receive a zero payoff with certainty. In the language of non-cooperative game theory, in this case a strategy in which a user chooses "neither option" is "weakly dominated" by the strategy in which that user chooses Option B. There is only one equilibrium in which no user selects a weakly dominated strategy. In this equilibrium, users 1 and 3 again select Option A, while all remaining users (i.e. users 2, 4, 5, 6, 7 and 8) select Option B. This outcome, summarized in Table 10 of Appendix B, is the worst of all Nash equilibrium outcomes, with total surplus equal to 386 and total efficiency equal to 50.1%.

3 Congestion Etiquettes

There are several approaches to addressing the Tragedy of the Commons problem.[7] One approach involves treating spectrum as a "private good" wherein a spectrum owner can employ market prices to ration spectrum among competing users. To the extent that this approach applies only to licensed spectrum owners, it fails to promote experimentation and innovation in spectrum usage. Another approach involves the use of administrative rules which restrict the technical characteristics of unlicensed radio equipment so that excessive use of unlicensed spectrum, in its common pool resource form, is an acceptably low probability.[8] In spite of such restrictions, excessive use of spectrum is still a distinct possibility. To address this problem, some unlicensed devices (e.g., wireless routers) employ congestion etiquettes, which are a set of rules that determine the set of users who can access spectrum, and the amount of spectrum assigned to each user.[9] Because there are a wide variety of rules, there are a wide variety of spectrum etiquettes.[10]

3.1 Enhancing Efficiency By Changing Strategy Payoff

We first consider an etiquette that addresses the congestion problem by changing the payoffs of users and, in so doing, their service choice. This etiquette is based on the observation that the Tragedy of the Commons problem is made worse by the fact that the

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[7] For a discussion of the Tragedy of the Commons problem related to real time access to spectrum, see Peha, Jon, " Spectrum Sharing Without Licenses: Opportunities and Dangers," in *Interconnection and the Internet: Selected Papers From the 1996 Telecommunications Research Conference*, G. Rosston and D. Waterman (Eds). Mahwah, N.J.: Lawrence Erlbaum Associates, 1997, pgs. 49-75.

[8] Technical restrictions such as power transmission limits on unlicensed devices limit the number of users that have access to the service and, thus, the likelihood of excessive spectrum use. To address signal interference, unlicensed devices employ filtering and error correction technologies to sort out the desired signal from other signals and background noise. The existence of such filters enables, for example, multiple Wi-Fi routers to co-exist within a given geographic area.

[9] It has been suggested that advances in technology are beginning to solve the congestion problem. For example, so-called "intelligent systems" are being developed that attempt to match available supply with demand and, in so doing, eliminate the amount of unused spectrum that exists at any moment. However, while this matching process is important, it doesn't address, in the presence of spectrum scarcity, whether the economically efficient set of matches are taking place.

[10] The only instance of an etiquette mandated by the FCC occurs in the Part 15 rules on unlicensed PCS. The etiquette, in the form of a "listen-before-talk" rule, is a spectrum-sharing technique that reduces the probability of conflicting signals by requiring the device to monitor for other spectrum signals for a fixed period of time before emitting its own signal. Unlicensed PCS devices must also limit transmission for a fixed duration. Due to the fact that unlicensed PCS was never widely adopted, the FCC changed its rules, amending the listen before talk rule and rendered this requirement all but moot.

strategy of selecting Option B weakly dominates the strategy of selecting neither Option A nor B. One method of changing user incentives is to charge users a small non-refundable fee if they select Option B. Herein called the "Pay to Connect" etiquette, a user who selects Option B, but who also expects that his/her decision would raise the demand for spectrum available under Option B to a level above his or her own congestion limit, would prefer to choose to sit on the sidelines and earn a zero payoff rather than to choose Option B and earn a negative payoff equal to the non-refundable fee.

Given a fee for selecting Option B, the strategies that sustain the inefficient Nash equilibrium reported above and in Table 10 of Appendix B no longer represent equilibrium strategies. Rather than select the free service while expecting to receive a negative payoff equal to the non-refundable fee, users 2, 4, 5, 6, 7 and 8 would each prefer to select neither option. In Example 1, the only Nash equilibrium strategy selection that survives the imposition of a small but positive fee for Option B is the highest surplus Nash outcome as described above and as reported in Table 11 of Appendix B. In this outcome total efficiency is equal to 96.5%.

3.2 Enhancing Efficiency Via Rationing – Randomization

We next consider two etiquettes that employ a randomization technique to more efficiently allocate spectrum to those users that select Option B. As before, users first decide whether to select Option A or Option B. For those who select Option B, a random number generator is used to assign a priority level to each such user. If n users select Option B, then there are $n!$ permutations of users, and each permutation is assumed to be chosen with equal probability.[11] Users are then assigned spectrum under Option B until the total demand exceeds a pre-determined level, which we assume to be equal to the highest congestion limit of all users who are expected to make use of the system.[12] Herein referred to as "Simple Randomization," this etiquette addresses the Tragedy of the

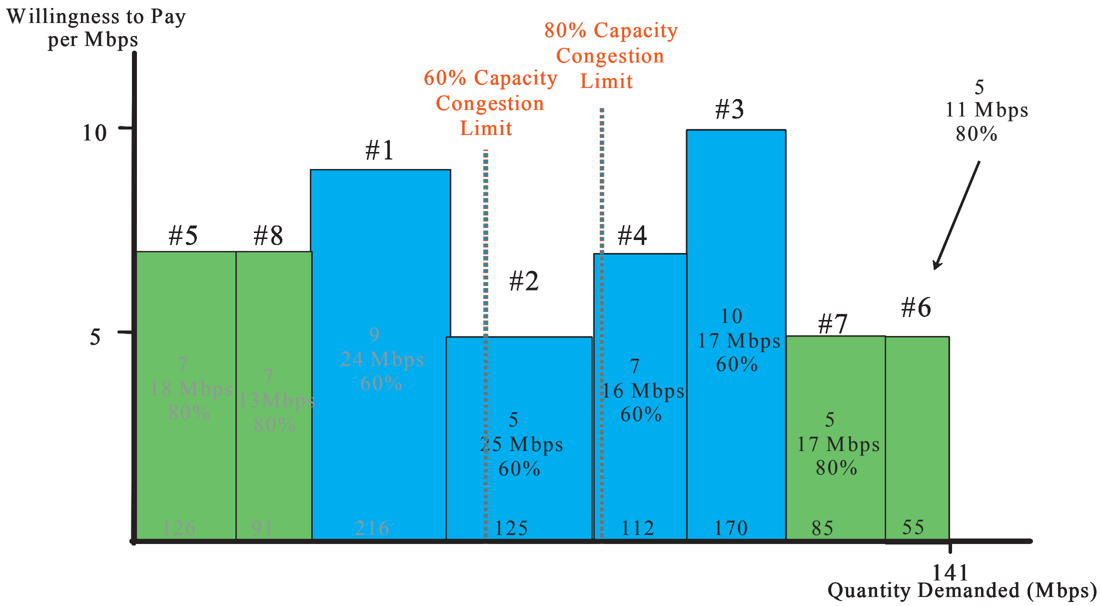
[11] When there are 8 users there are 40,320 possible permutations.

[12] In an alternative version of this randomization etiquette, users could be served only until total demand exceeds the lowest congestion limit of expected users. There are clear trade-offs involved in selecting the appropriate system capacity, and neither rule is optimal in all situations. With a high limit, some high priority users with low congestion limits would be granted service under the etiquette, but the service would be worthless to them if the total demand exceeded their personal congestion limit. On the other hand, there can be situations in which some users would be served when system capacity is set at the high congestion limit but would be rejected under the lower limit.

Commons problem by guaranteeing that some users (those with high priority and high congestion limits) receive some value from their assigned spectrum under Option B. The algorithm which underlies the above approach is known as a "greedy algorithm." The algorithm addresses the congestion problem by evaluating, in a sequential fashion, each user's spectrum demand. The term "greedy" refers to the "take what you can get now" strategy inherent in its solution rule. The failure to consider the demands of all users simultaneously may result in a solution value that falls far short of the global optimum.[13]

A particular implementation of the simple randomization etiquette for Example 1 is illustrated in Figure 3 below. In this case, all 8 users are assumed to select Option B, and the random priority assignment ranks users in the order {5, 8, 1, 2, 4, 3, 7, and 6}.

Figure 3: Simple Randomization in Example 1



Under this etiquette, users 5, 8, 1, and 2 are provided spectrum, while all lower priority users are excluded, because including any additional user would raise total

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[13] For example, it is possible that global welfare optimization considerations might require that a high priority user, who is initially guaranteed service, should later be excluded in order for one or more higher valued, but lower priority users, to be served.

demand above the system capacity of 80, which is equal to the maximum congestion limit. In this case, users 1 and 2 would receive no value from their spectrum assignment since the total demand would exceed their personal congestion limits, and the resulting total efficiency would be equal to 16.3%.[14] Alternative user selections between Options A and B will result in different outcomes under the Simple Randomization etiquette. In fact, there is a unique Nash equilibrium in this case whose outcome is shown in Appendix B. In this equilibrium, total efficiency is equal to 83.6%.

In an alternative version of the randomization etiquette, to be called "Informed Randomization", users are first asked to report their personal congestion limits to a system manager.[15] As in simple randomization, a random number generator is used to assign a priority level to each user who selects Option B. Service is now assigned, however, by taking account of the congestion limits of each potential user. If the demand of the highest priority user is less than that user's congestion limit, that user is guaranteed service. The etiquette then considers each additional user's demand and personal congestion limit in order of decreasing priority. The etiquette ensures that total demand always remains less than or equal to the minimum congestion limit of all users who are guaranteed service. With each new user, the relevant system congestion limit is set equal to the minimum limit of the current user being considered, and the limits of all higher ranking users already guaranteed service. If the total demand, including the current user, is less than or equal to the congestion limit, that user is granted service. Otherwise, that user is not served and the next highest priority user is considered. The etiquette continues to examine users until all have been considered, or until demand is exactly equal to the system limit as determined by the currently served users.

[14] The sensitivity of the randomization (and later) etiquettes to personal congestion limits raises the question of whether users will have the incentive to report their limits truthfully. In fact, "truth telling" is in this case is a weakly dominant strategy (in game theoretic terminology) since a user who selects Option B can never increase his or her payoff by falsely reporting a congestion limit. To see this, note that a high priority user with a low (e.g. 60%) congestion limit would never wish to report a higher limit. If the user would have been excluded under a truthful (60%) report, while a false (e.g., 80%) report might result in receiving service, the service would be of no value to the user. Similarly, a user with a high congestion limit would never wish to report a lower limit, since in some cases this would result in that user being denied service, while in no cases would a lower total demand increase the quality of service under the present assumptions.

[15] The system manager would most likely be embodied in a hardware device, and user reports about personal congestion limits would correspond to priority bits under existing internet protocols.

Under the Informed Randomization Etiquette, users 2, 4, 5, 6, 7, and 8 would expect to receive positive surplus under Option B, since there is positive probability that they will be assigned a high priority rank. They will therefore choose Option B in preference to choosing neither Option A nor Option B. In addition, user 3 in this example has an incentive to select Option B over Option A, because the *expected* surplus to him or her under the randomization protocol is higher than could be achieved *with certainty* by choosing Option A. For user 1, the certain surplus under Option A is higher than that under the randomization etiquette, whether or not user 3 chooses Option B. The resulting outcome is shown in Appendix B.

While the informed randomization etiquette guarantees that those users who select Option B, and who have a sufficiently high priority level, receive positive surplus, this protocol does not necessarily create the correct incentives for users to make optimal selections between Options A and B. In particular, by raising a high valued user's expected return from selecting Option B, the randomization etiquette may induce such a user to select Option B, a choice that while private welfare maximizing for the user, ends up reducing total surplus. In this example, the total surplus under the informed randomization etiquette is equal to 560.25 and the corresponding efficiency is equal to 72.8%.

3.3 Enhancing Efficiency Via Rationing – Willingness to Pay

We next consider etiquettes that utilize reported information on a user's congestion tolerance and willingness to pay to more efficiently allocate spectrum to those users that select Option B.[16] In one version of a WTP etiquette (hereafter called the "Lump Sum WTP Etiquette"), users who select Option B would be asked to report both their personal congestion limit and their willingness to pay to receive or send information. The reported

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[16] In his article "Beyond Spectrum Auctions", Eli Noam, Beyond Spectrum Auctions: Taking the Next Step to Open Access" Journal of Law and Economics Vol. 41 pp. 465 (1998). Noam proposes a system of open access to replace the current auction-based licensing system. Under the proposed approach spectrum users pay an access fee based on the level of congestion in frequency bands at a particular time. Under this arrangement, users would employ intelligent agents to pay for their spectrum consumption using electronic tokens and a set of clearing houses. A similar approach was proposed by Lehr (2004). See, William Lehr, "Economic Case for Dedicated Unlicensed Spectrum Below 3GHz," page 19, Unpublished Manuscript. Our WTP approach differs in several important respects to these approaches. The above approach attempts to solve the spectrum allocation problem conditional on the existing level of spectrum congestion. In contrast, our approach attempts to solve the spectrum allocation problem on an unconditional basis.

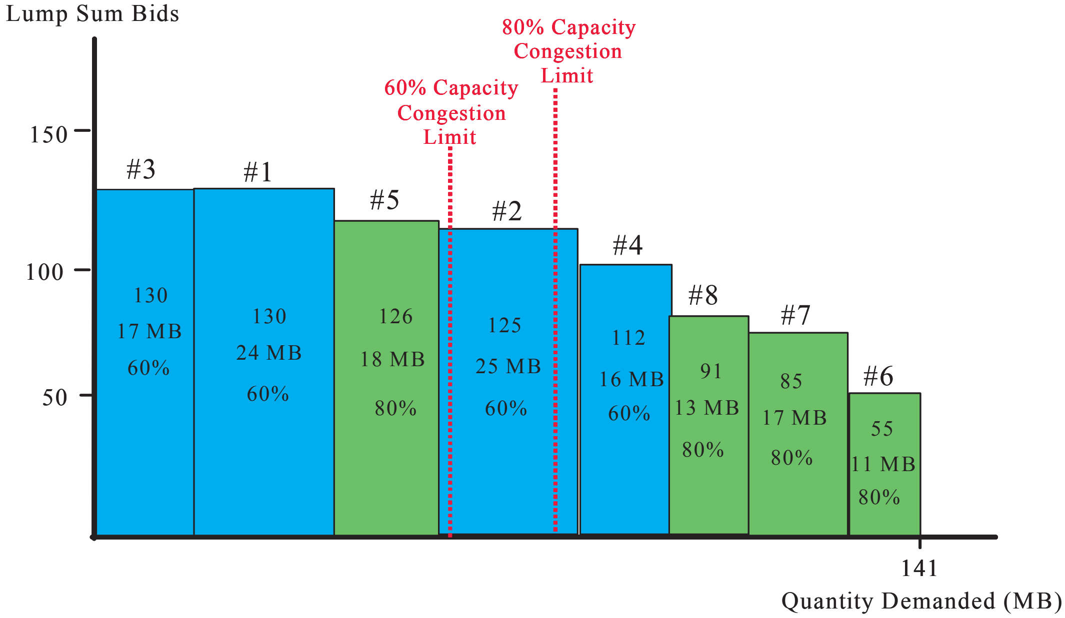
willingness to pay values would then be used to define a priority ranking of users, and the etiquette would proceed in a similar manner as the randomization etiquette (but without the random component). In one important difference, however, users under the WTP etiquette would be required to actually pay a price to receive service. Under this etiquette, the price is determined by the reported willingness to pay of the marginal user who is excluded from service by the etiquette. Only users who are guaranteed service under the etiquette are required to pay this price.[17]

For example, suppose that in Example 1, all users bid "truthfully" in the following sense: users 1 and 3 bid 130, which is the price they would have paid if they had selected Option A, and all remaining users bid their actual value, which in each case is less than 130.[18] Users 1 and 3 would then be tied for the highest priority of service, and they would be followed in order by users 5, 2, 4, 8, 7 and 6. This WTP etiquette employs a greedy algorithm to solve the congestion problem. In particular, users 1, 3 and 5 would therefore be assigned spectrum under Option B and would each pay a price equal to 125, which is the highest rejected bid. No other users will be assigned spectrum, since the addition of any other user would violate the requirement that total demand remain less than or equal to the minimum congestion limit of all users who are guaranteed service.

[17] This pricing rule is based on standard "second price" auction theory. In the present context, there is some ambiguity about the definition of the marginal user. The present version of the etiquette assumes that the extra-marginal user is defined as the highest priority user who is excluded from service, and such that no lower priority user is assigned service.

[18] In general, truthful bidding of this form is not guaranteed. While truthfully revealing a personal congestion limit is still a weakly dominant strategy, there can be situations in which users may prefer to misrepresent their willingness to pay in an effort to lower the market price.

Figure 4: Willingness to Pay in Example 1



The ability of users to express their willingness to pay introduces some surprising and important strategic effects. For example, users with high valuations can now predict with greater certainty whether or not they will be served if they choose Option B. No user who can afford to pay the subscription price for Option A can gain by choosing Option B and making a bid higher than the subscription price for Option A (for which satisfactory service is guaranteed).[19] Assuming that all users place "truthful" bids as described above, in the set of undominated Nash equilibria for Example 1, reported in Appendix B, all users select Option B. In this equilibrium both total surplus and consumer surplus are lower than surplus under both of the randomization etiquettes and under the Pay to Connect etiquette. Total efficiency under this WTP etiquette for Example 1 is equal to 66.5%.

3.4 Congestion Etiquettes – Nash Predictions

The full set of Nash equilibrium predictions for Example 1 and an additional example are summarized in Tables 1 and 2 below, and full results are presented in Appendix B. Efficiency is defined as the surplus earned by both spectrum users and

[19] A bid for service under Option B that is higher than the price of Option A would differ from a bid equal to the price of Option A only if the bid of the marginal user under Option A was higher than the price of Option A. In such a case, the bidder would be better off choosing Option A instead of Option B.

service providers under the corresponding etiquette, as a fraction of maximum possible surplus, expressed in percentage terms, while consumer surplus measures the amount of surplus enjoyed by spectrum users. Each row reports only the equilibrium efficiency associated with un-dominated strategies under the corresponding etiquette. Examples 1 and 2 refer to different user valuation environments. They differ primarily in terms of the set of users who can afford to purchase the subscription service under Option A. In Example 1, two out of eight users can afford Option A as illustrated in Figure 1, while in Example 2 user demands and values of service are changed in such a way that five out of eight users can afford Option A.

In Example 1, each of the four pro-active etiquettes achieves higher efficiencies than that achieved under the fairness etiquette, with the pay to connect etiquette achieving by far the best performance in terms of both efficiency and consumer surplus. In Example 2, Nash equilibrium theory predicts a benign outcome for the fairness etiquette suggesting the absence of a Tragedy of the Commons problem in this case. Now, two of the pro-active etiquettes (pay to connect and simple randomization) achieve outcomes close or equal to the default outcome, while the remaining two etiquettes (informed randomization and willingness to pay) achieve significantly worse outcomes.

Table 1: **Summary of Predicted Efficiencies**

Etiquette	Example 1	Example 2
Fairness	50.1%	91.4%
Pay to Connect	96.5%	91.4%
Simple Randomization	83.6%	91.4%
Informed Randomization	72.8%	54.7%
WTP	66.5%	76.9%

Table 2: Summary of Predicted Consumer and Total Surplus Values

Etiquette	Example 1		Example 2	
	Total Surplus	Consumer Surplus	Total Surplus	Consumer Surplus
Fairness	386	126	1018	628
Pay to Connect	743	363	1018	528
Simple Randomization	643.367	383.367	1018	628
Informed Randomization	560.25	430.25	609.593	479.593
WTP	512	137	857	309

4 Experimental Analysis – Congestion Etiquette Performance

Eight subjects are assigned a set of characteristics, including the value they place on having the ability to transmit or receive information, the minimum amount of bandwidth he/she needs to transmit or receive such information, and a "congestion limit," expressed as the maximum amount of spectrum congestion the user can tolerate before the value he/she places on employing spectrum falls from some desired value to zero. To test the sensitivity of the results to changes in the assigned characteristics, two different valuation environments were created. One environment contained the valuations shown in Example 1.

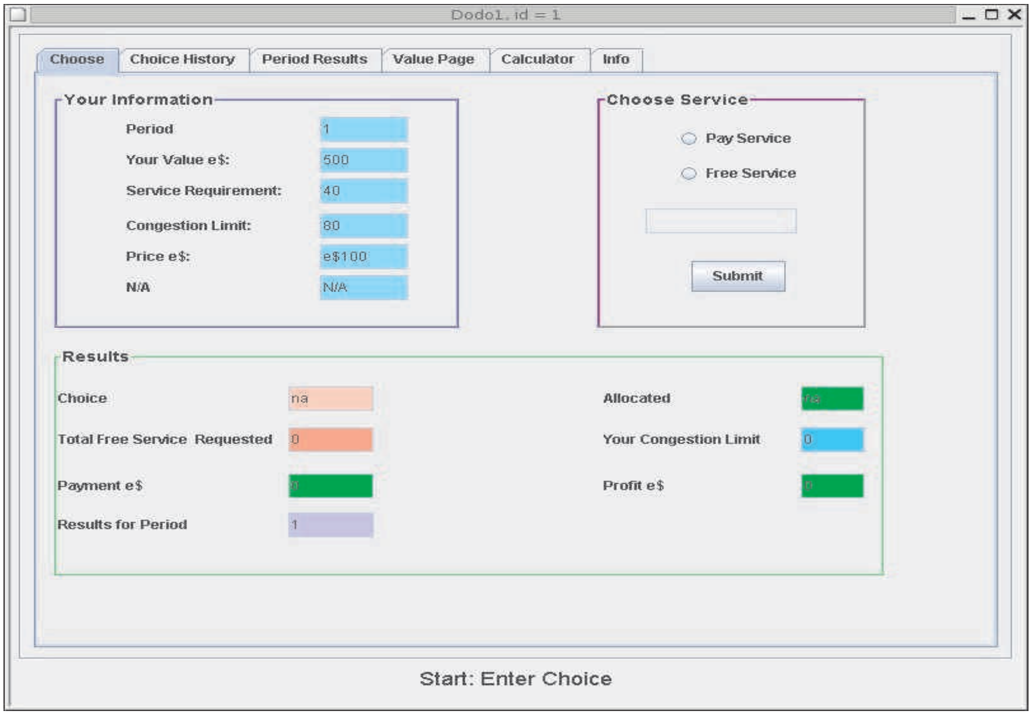
Each subject had the option to choose between a reliable and non-congestible transmission system that requires a fixed payment of 130 units (Option A), and a transmission system that is "free," but where the value to each user depends on the congestion level of the free service's transmission system (Option B). The capacity of the free service is assumed to be equal to 100 units. If two or more subjects choose Option B, depending on the total amount of bandwidth demanded, a congestion etiquette may be employed that determines how much spectrum each subject is assigned and the value each subject obtains from their bandwidth assignment. The etiquettes fall into two broad categories – etiquettes that do not exclude users and etiquettes that employ algorithms to exclude users.

4.1.1 No Exclusion Etiquette – Fairness and Pay to Connect

Under a no exclusion etiquette, spectrum is allocated to all users independent of the total amount of bandwidth demanded. However, depending on the total amount of bandwidth demanded, the assigned spectrum may translate to a transmission speed that is unacceptable to some users given their desired service application (e.g., streaming video). If total demand results in a congestion level in excess of 80%, then no user obtains any value from their allocated spectrum. If the total amount of bandwidth demanded results in a congestion level greater than 60%, but less than 80%, only those users who are tolerant of high congestion receive value from their allocation. Finally, if total amount of bandwidth demanded results in a congestion level less than or equal to 60%, all users receive value from their spectrum allocation. Two types of no exclusion etiquettes were tested. Under the fairness etiquette, there is nothing in the economic environment, other than the desire of subjects to coordinate their service option choices, which addresses the spectrum congestion problem. Under the Pay to Connect etiquette, a small fee placed on the "free" service is designed to discourage some subjects from selecting that option.

Figure 5 presents the electronic interface subjects used to make their option choice. The interface provides each subject personalized information on the value he/she places on accessing spectrum, the amount of spectrum the he/she needs, and the subject's spectrum congestion limit. Each participant is asked to choose between a subscription service (Option A) and a free service (Option B). In instances where the subject chooses Option B, the interface also provides amount of spectrum actually assigned to the subject given the choices made by other subjects.

Figure 5: Screen 1 – No Exclusion



4.1.2 Exclusion Etiquettes

We examine the performance properties of four different market-informed etiquettes that exclude users. Three of the etiquettes employ a greedy algorithm to exclude users, while one etiquette excludes users by solving for the efficient allocation of bandwidth across prospective users based on their reported willingness to pay and congestion limits.

4.1.2.1 Randomization Etiquette

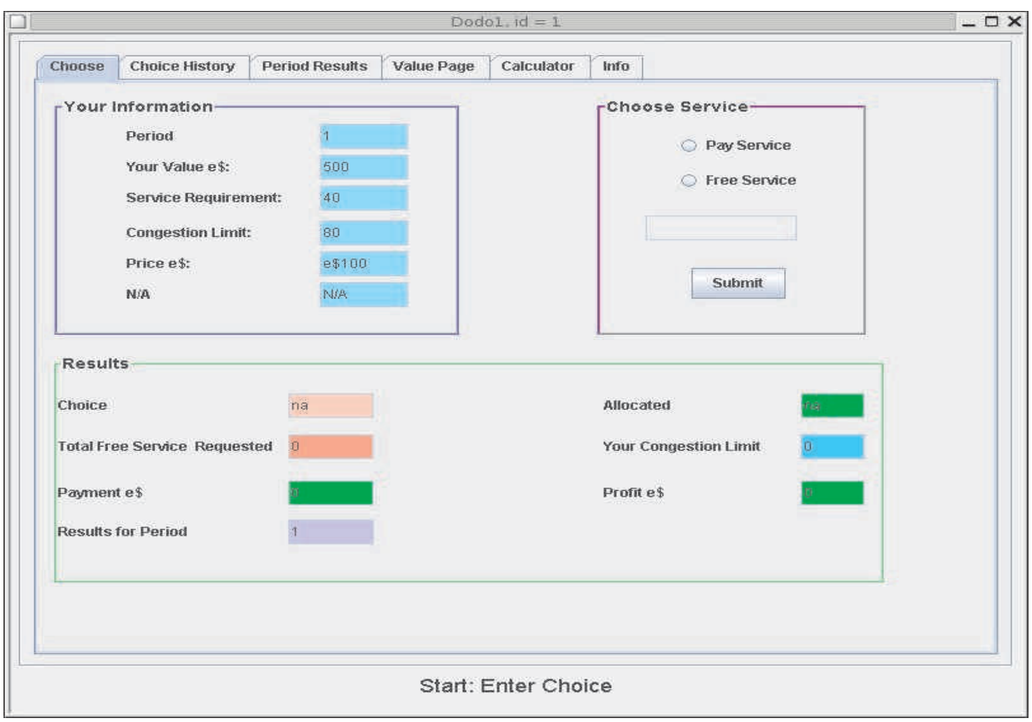
In the randomization etiquette, the subjects first choose between Options A and B. Those who choose Option B report their bandwidth congestion limits and are randomly assigned a priority number.[20] The etiquette ranks all subjects based on this random priority assignment. Given the size of existing capacity, relative to the total demand of all subjects choosing Option B, the highest ranked subject is assigned bandwidth provided that his or her demand is less than their congestion limit. The subject with the

[20] In a technical appendix, it is demonstrated that truthful reporting is a dominant strategy for each experimental subject. In any case, the algorithm behind allocates spectrum to users *as if* all subjects have reported truthfully.

next highest priority ranking is allocated spectrum if the total demand of the two subjects is less than the minimum congestion limit associated with both subjects. The etiquette proceeds by evaluating, in a sequential fashion whether subjects with successively lower rank should be assigned bandwidth. At every stage, if total bandwidth demand is less than the relevant congestion limit (i.e., minimum congestion of all subjects who are assigned spectrum), then that user is assigned bandwidth. If a user's demand exceeds the relevant congestion limit, then the protocol rejects that user and it goes on to evaluate the bandwidth needs of the next lowest ranked user. This process continues until all users have been examined.

Figure 6 presents the electronic interface subjects used to express their option choice. The interface provides each subject personalized information on the value he/she places on accessing spectrum, the amount of spectrum the he/she needs, and the subject's spectrum congestion limit. Each participant is asked to choose between a subscription service (Option A) and a free service (Option B). In instances where the subject chooses Option B, the interface also provides amount of spectrum actually assigned to the subject given the choices made by other subjects.

Figure 6: Screen 1 – Greedy Algorithm (Random)

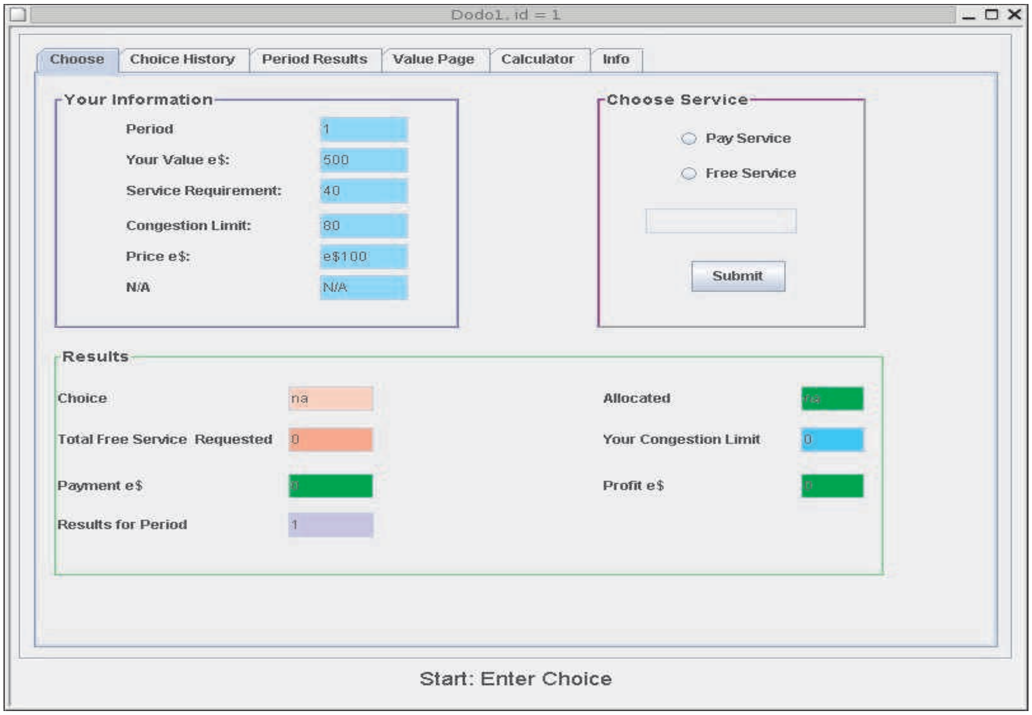


4.1.2.2 Lump Sum WTP-based Greedy Algorithms

In this etiquette, the subjects first choose between Options A and B. Those who choose Option B report their congestion limits and submit a lump sum bid that represents the amount of money they are willing to pay in order to be assigned a priority level for service. The etiquette ranks all subjects from the highest to the lowest rank, based on the size of the reported lump sum bids. The algorithm then proceeds exactly as the randomization etiquette. The highest ranked subject is assigned bandwidth if his or her demand is less than or equal to his or her congestion limit. The second ranked subject is allocated bandwidth if the total demand of the two subjects is less than the minimum congestion limit associated with those subjects. The etiquette proceeds by evaluating, in a sequential fashion, whether subjects with successively lower rank should be assigned bandwidth, and continues until all users have been examined.

Figure 7 presents the electronic interface subjects used to make their option choice. The interface provides each subject its assigned information on the value he/she places on accessing spectrum, the amount of spectrum he/she needs, and the subject's spectrum congestion limit. Each participant is asked to choose between a subscription service (Option A) and a free service (Option B). If Option B is selected, the subject is asked to submit the maximum amount of money, on a lump sum basis, he/she is willing to pay in order to access its desired amount of spectrum. In instances where the subject chooses Option B, the interface also provides the amount of spectrum, if any, that is actually assigned to the subject given the choices made by other subjects. Finally, the interface identifies the market price the subject pays by accessing spectrum through Option B.

Figure 7: Screen 1 – Greedy Algorithm (Lump Sum)

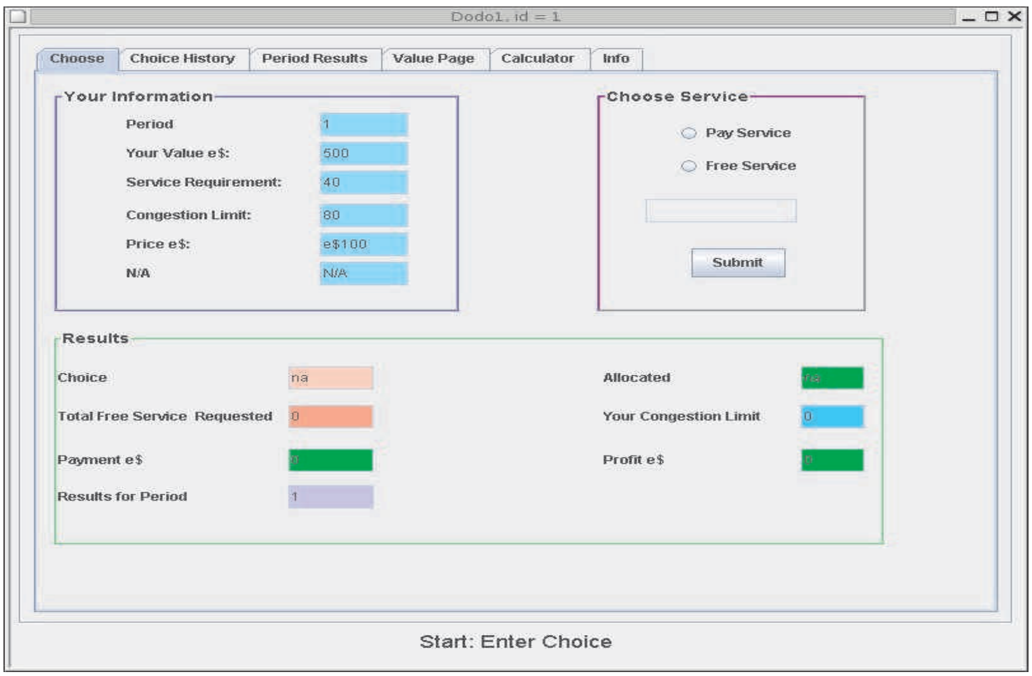


4.1.2.3 Unit-based WTP-based Greedy Algorithms

In this etiquette, the subjects first choose between Options A and B. Those who choose Option B report their congestion limits and a bid that represents the amount of money they are willing to pay, on a per megabit basis, in order to be assigned a priority level for service. The etiquette ranks all subjects from the highest to the lowest rank, based on the size of the per unit bids. The algorithm then proceeds exactly as the randomization etiquette. The highest ranked subject is assigned bandwidth if his or her demand is less than or equal to his or her congestion limit. The second ranked subject is allocated bandwidth if the total demand of the two subjects is less than the minimum congestion limit associated with those subjects. The etiquette proceeds by evaluating, in a sequential fashion, whether subjects with successively lower rank should be assigned bandwidth, and continues until all users have been examined.

Figure 8 presents the electronic interface subjects used to make their option choice. The interface provides each subject its assigned information on the value he/she places on accessing spectrum, the amount of spectrum he/she needs, and the subject's spectrum congestion limit. A subject makes a choice by selecting whether to access spectrum either through a pay service (Option A) or a free service (Option B). If Option B is selected, the subject is asked to submit the maximum amount of money, on a per unit basis, he/she is willing to pay in order to access its desired amount of spectrum. In instances where the subject chooses Option B, the interface also provides the amount of spectrum, if any, that is actually assigned to the subject given the choices made by other subjects. Finally, the interface identifies the market price the subject pays by accessing spectrum through Option B.

Figure 8: Screen 1 – Greedy Algorithm (Per Unit)

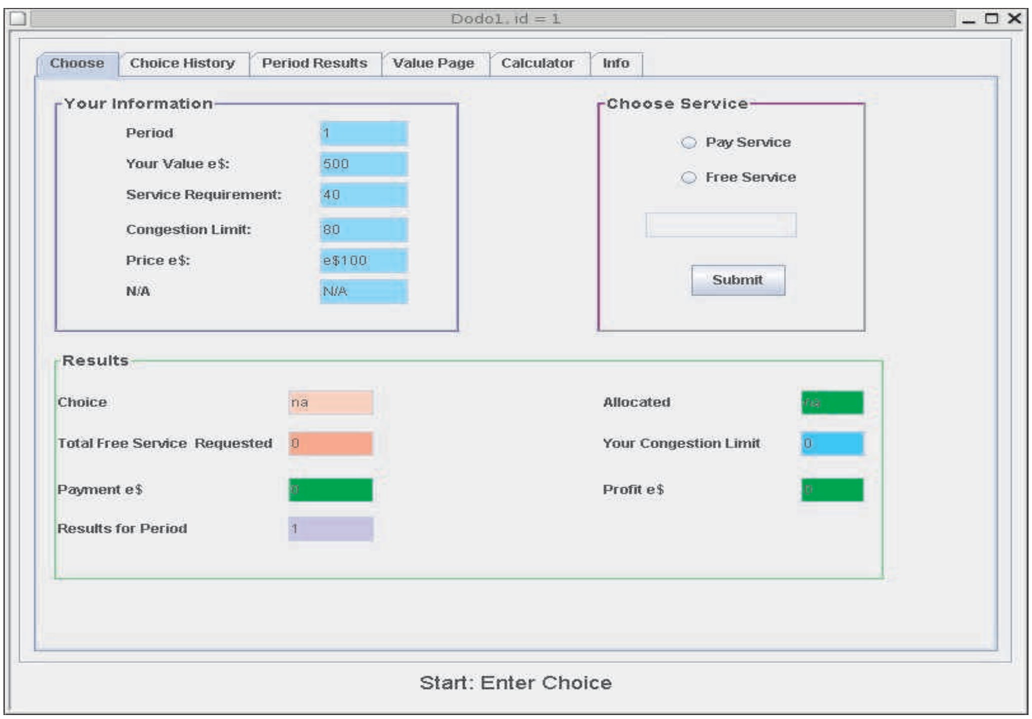


4.1.2.4 WTP-based Full Optimization Algorithm

In this etiquette, the subjects first choose between Options A and B. Those who choose Option B report their congestion limits and submit a lump sum bid that represents the amount of money they are willing to pay in order to be assigned a priority level for service. A mathematical algorithm then evaluates all the different ways in which the users can be assigned bandwidth and identifies that assignment which maximizes the total social value of spectrum, given the system capacity of Option B and the demand and reported congestion limits of each user.

Figure 9 presents the electronic interface subjects used to make their option choice. The interface provides each subject its assigned information on the value he/she places on accessing spectrum, the amount of spectrum he/she needs, and the subject's spectrum congestion limit. A subject makes a choice by selecting whether to access spectrum either through a pay service (Option A) or a free service (Option B). If Option B is selected, the subject is ask to submit the maximum amount of money, on a per unit basis, he/she is willing to pay in order to access its desired amount of spectrum. In instances where the subject chooses Option B, the interface also provides the amount of spectrum, if any, that is actually assigned to the subject given the choices made by other subjects. Finally, the interface identifies the market price the subject pays by accessing spectrum through Option B.

Figure 9: Screen 1 – Full Optimization



4.2 Experimental Results

The results of the experimental investigation are shown in Tables 3 – 4. As shown in Table 3, depending on the valuation environment, society captures only between 42% and 57% of the gains that are available from its spectrum resource when the Fairness Etiquette is used to address spectrum congestion involving unlicensed use. Results also show that, in both valuation environments, the average efficiency under the Fairness Etiquette is consistently less than the average efficiency achieved by the other examined etiquettes. For example, the Full Optimization Etiquette achieves average efficiency levels across the two environments that are consistently and substantially higher than the efficiency levels achieved by the Fairness Etiquette. The results therefore indicate that society can experience a substantial increase in its welfare from using the Full Optimization Etiquette to handle spectrum congestion involving unlicensed use. Interestingly, as shown in Table 3, each member of the class of new, market-informed etiquettes performed better than the Fairness Etiquette. One interesting result is the performance achieved by the Pay to Connect Etiquette. If the Greedy Algorithm-based etiquettes represent high-tech solutions to the wireless congestion problem, the Pay to Connect Etiquette is the decidedly low-tech solution. Despite its low-tech nature, the Pay to Connect Etiquette's performance compares favorably to the performances achieved by the Greedy Algorithm-based etiquettes.

The experimental results provide inconsistent support for the belief that spectrum users behave in a manner that is consistent with Nash Equilibrium Theory as measured by market efficiency. As shown in Table 3, while Nash Predictions regarding efficiency are close to the efficiency levels observed in the experiments for the Informed Randomization and Lump Sum –WTP Etiquettes, the theory does not accurately predict the efficiency performance of the Fairness and the Pay to Connect Etiquettes. The mixed performance of Nash Equilibrium Theory in predicting outcomes is also observed with regards to consumer surplus outcomes. As shown in Table 4, while Nash Predictions regarding consumer surplus are close to the consumer surplus levels observed in the experiments for the Informed Randomization Etiquette, the theory does not accurately predict the consumer surplus outcomes generated under the Pay to Connect Etiquette.

Because the market-informed etiquettes use market prices to address the spectrum congestion problem, it is useful to assess how spectrum users fair under these etiquettes versus the Fairness Etiquette. Experimental results shown in Table 4 shed light on this issue. According to the results, with the exception of the Pay to Connect Etiquette involving the second valuation environment, consumers are much better off under the proposed market-informed congestion etiquettes than under the Fairness Etiquette. In particular, despite paying a fee at times of excessive congestion, spectrum users are better off under these market-informed etiquettes than under the Fairness Etiquette. The reason for this is straightforward. The spectrum congestion problem is really two problems in one. Because of the heterogeneous needs of spectrum users, society obtains the highest value from its spectrum resource only if: (1) spectrum is being employed by the highest valued users and, (2) the "right" quality of service is being provided. Solving this problem involves obtaining information on the valuation each users places on sending and receiving information and their congestion limit. Each member of the class of new etiquettes incorporates this information, in varying degrees, in addressing the spectrum congestion problem and, in so doing, identifies a more efficient assignment of users to spectrum. This greater efficiency makes it possible for individual users to be better off under these new etiquettes than under the Co-Existence Etiquette, despite paying a fee in the presence of excessive congestion.

Table 3: Experimental Results and Nash Predictions: Mean Efficiency

<table>
<tr><th colspan="3" rowspan="2">Etiquette</th><th colspan="2">Example 1</th><th>Nash Predictions</th><th colspan="2">Example 2</th><th>Nash Predictions</th></tr>
<tr><th>N</th><th>Efficiency</th><th>Efficiency</th><th>N</th><th>Efficiency</th><th>Efficiency</th></tr>
<tr><td colspan="3">Fairness</td><td>18</td><td>42%</td><td>50.1%</td><td>22</td><td>57%</td><td>91.4%</td></tr>
<tr><td colspan="3">Pay to Connect</td><td>27</td><td>52%</td><td>96.5%</td><td>33</td><td>74%</td><td>91.4%</td></tr>
<tr><td rowspan="3">Greedy Algorithm</td><td colspan="2">Informed Randomization</td><td>27</td><td>70%</td><td>72.8%</td><td>33</td><td>63%</td><td>54.7%</td></tr>
<tr><td rowspan="2">Willingness to Pay</td><td>Lump Sum</td><td>27</td><td>72%</td><td>66.5%</td><td>33</td><td>68%</td><td>76.9%</td></tr>
<tr><td>Unit Bid</td><td>27</td><td>66%</td><td>-</td><td>33</td><td>74%</td><td>-</td></tr>
<tr><td colspan="3">Full Optimization</td><td>27</td><td>75%</td><td>-</td><td>33</td><td>73%</td><td>-</td></tr>
<tr><td colspan="3">Total</td><td>150</td><td>64%</td><td>-</td><td>190</td><td>69%</td><td>-</td></tr>
</table>

Table 4: Experimental Results and Nash Predictions: Mean Consumer Surplus

<table>
<tr><th colspan="3" rowspan="2">Etiquette</th><th colspan="2">Example 1</th><th>Nash Predictions</th><th colspan="2">Example 2</th><th>Nash Predictions</th></tr>
<tr><th>N</th><th>Consumer Surplus</th><th>Consumer Surplus</th><th>N</th><th>Consumer Surplus</th><th>Consumer Surplus</th></tr>
<tr><td colspan="3">Fairness</td><td>18</td><td>120</td><td>126</td><td>22</td><td>260</td><td>628</td></tr>
<tr><td colspan="3">Pay to Connect</td><td>27</td><td>62</td><td>363</td><td>33</td><td>260</td><td>528</td></tr>
<tr><td rowspan="3">Greedy Algorithm</td><td colspan="2">Informed Randomization</td><td>27</td><td>390</td><td>430.25</td><td>33</td><td>470</td><td>479.59</td></tr>
<tr><td rowspan="2">Willingness to Pay</td><td>Lump Sum</td><td>27</td><td>250</td><td>137</td><td>33</td><td>280</td><td>309</td></tr>
<tr><td>Unit Bid</td><td>27</td><td>270</td><td>-</td><td>33</td><td>340</td><td>-</td></tr>
<tr><td colspan="3">Full Optimization</td><td>27</td><td>210</td><td>-</td><td>33</td><td>300</td><td>-</td></tr>
<tr><td colspan="3">Total</td><td>150</td><td>-</td><td>-</td><td>190</td><td>-</td><td>-</td></tr>
</table>

4.3 Concluding Comments

This study has employed both theoretical and experimental methods to evaluate a number of congestion etiquettes which can potentially address the Tragedy of the Commons problem with respect to spectrum congestion. The examined etiquettes fall into two categories. One class of etiquette attempts to solve the congestion problem by changing the payoffs that some users receive by selecting Option B. In particular, under the Pay to Connect Etiquette, users pay a small fee for selecting Option B, and have an incentive to refrain from selecting this option (and therefore avoid creating a negative externality on other users) when it is expected that their decision would lead to an outcome that is both individually non-rational and globally inefficient. Thus, the Pay to Connect option can, in theory, lead to superior choices between Options A and B, but at the same time, it does not lead to efficient spectrum allocation among the users who finally choose Option B.[21]

Another class of etiquettes attempts to more efficiently allocate spectrum among those users that choose Option B. Both the Simple and Informed Randomization Etiquettes work by explicitly denying service to low priority users when total demand threatens to exceed a pre-defined system congestion limit. However, randomization has a potentially perverse effect on user choices between Options A and B. On the one hand, it gives every user who cannot afford to choose the subscription service (i.e., Option A) a positive incentive to choose Option B, rather than choose neither option. It can also give some users an incentive to switch from Option A to Option B, in cases where their expected payoff in Option B is higher than the certain payoff in Option A. The latter effect is more pronounced under the informed version of the etiquette than under the simple version. As with the Pay to Connect Etiquette, an etiquette based on the randomized ranking of users does not ensure that the spectrum available under Option B is assigned to those users that value such spectrum the most.

The Willingness to Pay etiquette has both potential advantages and potential disadvantages compared to the randomization etiquettes. User willingness to pay can potentially allocate service to the highest value users who choose Option B. However, high value users (users who would be willing to pay the subscription price for Option A) can reduce the uncertainty regarding whether they will obtain service via Option B by submitting a bid that is equal to the cost of accessing spectrum via Option A. As a result of this reduction in uncertainty, more high value users now have an incentive to select Option B, and these decisions can potentially reduce total surplus by leading to more potential congestion under Option B.

[21] In general, Pay to Connect does not result in a unique equilibrium, or guarantee that the highest equilibrium surplus is attained.

Appendix A: Figures and Charts

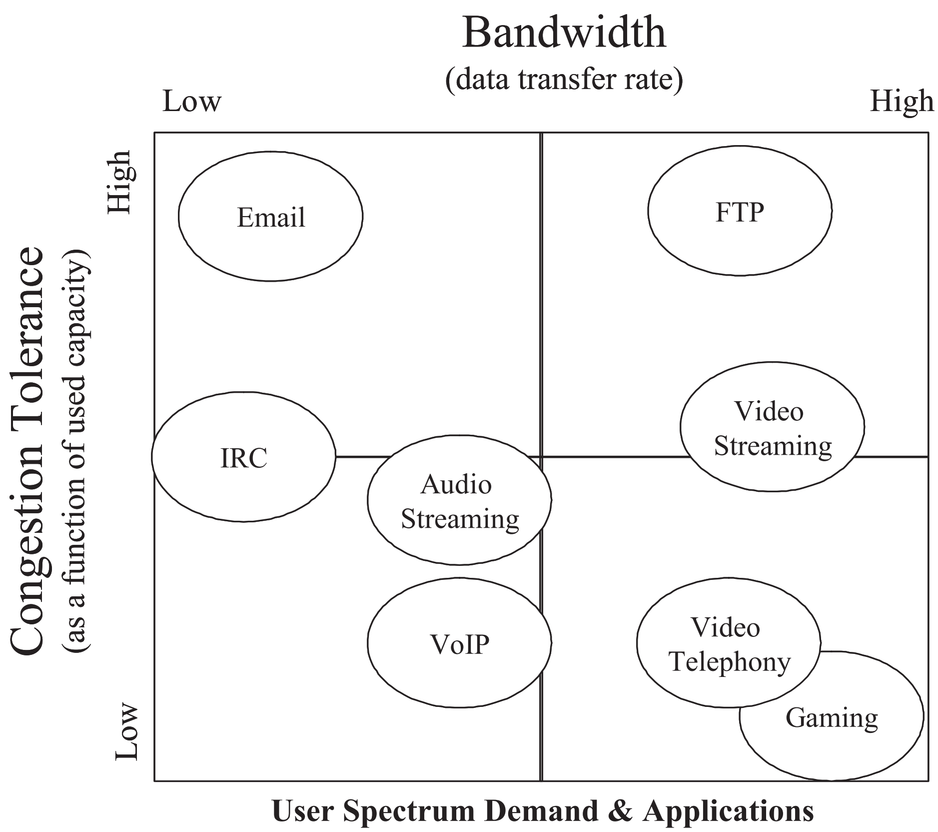
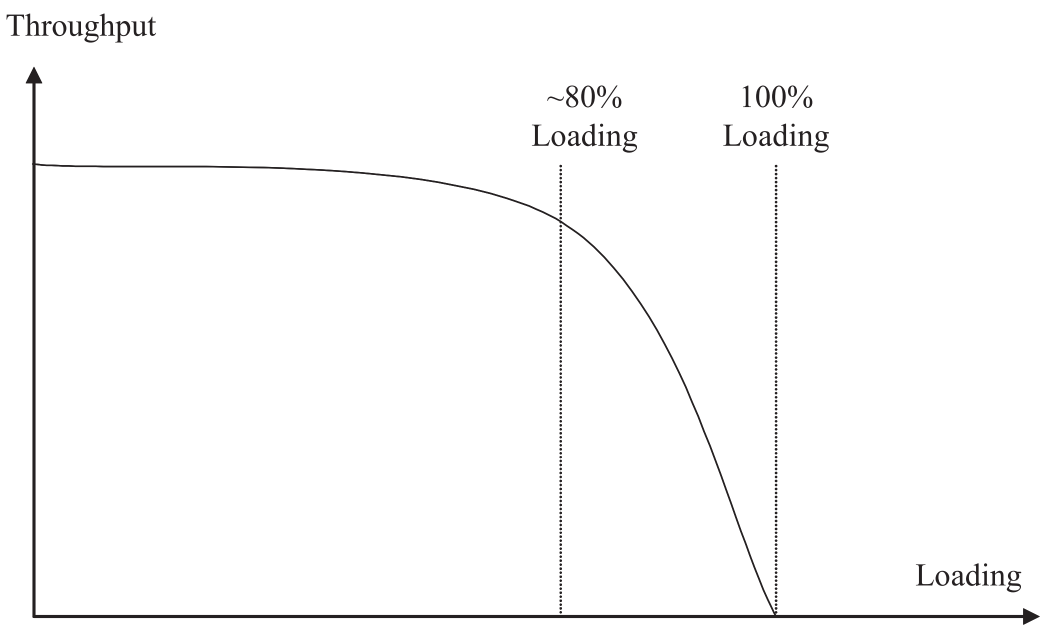


Figure A1.

Figure A2. Ethernet Packet Loss Function



Appendix B: Detailed Data from Examples

Table 9: Parameters for Example 1

Price of Option A: 130 Capacity of Option B: 100 Total Surplus Max: 770 Consumer Surplus Max: 528				
User (i)	Value per Unit (v_i)	Unit Demand (q_i)	Surplus (v_i q_i)	Congestion Limit
1	9	24	216	60
2	5	25	125	60
3	10	17	170	60
4	7	16	112	60
5	7	18	126	80
6	5	11	55	80
7	5	17	85	80
8	7	13	91	80

Table 10: Undominated Nash Equilibrium Assignments for Example 1 Assuming Co-Existence[22]

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 3	2, 4, 5, 6, 7, 8		386	126

Table 11: Nash Equilibrium Assignment for Example 1 Assuming Pay to Connect Etiquette (Price of B = 1)

Option A	Option B	Neither Option	Total Surplus	Consumer Surplus
1, 3	5, 6, 7, 8	2, 4	743	479

Table 12: Nash Equilibrium Assignment for Example 1 Assuming the Simple Randomization Etiquette

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 3	2, 4, 5, 6, 7, 8		643.367	383.367

Table 13: Nash Equilibrium Assignment for Example 1 Assuming Congestion Informed Randomization Etiquette

Option A	Option B	Neither Option	Total Surplus	Consumer Surplus
1	2, 3, 4, 5, 6, 7, 8		560.25	430.25

[22] There are 18 Nash equilibria altogether. Total surplus ranges from 386 to 743. Consumer surplus ranges from 126 to 483.

Table 14. Undominated Nash Equilibrium Assignments for Example 1 Assuming Lump-Sum Willingness to Pay Etiquette

Option A	Option B	Neither Option	Total surplus	Consumer surplus
	1, 2, 3, 4, 5, 6, 7, 8		512	137

Table 8: Parameters for Example 2

Price of Option A: 130	Capacity of Option B: 100			
Total Surplus Max: 1114	Consumer Surplus Max: 628			
User (i)	Value per Unit (v_i)	Unit Demand (q_i)	Surplus (v_i q_i)	Congestion Limit
1	8	19	152	60
2	10	17	170	60
3	6	16	96	60
4	10	22	220	60
5	7	13	91	80
6	8	18	144	80
7	9	19	171	80
8	5	14	70	80

Table 15: Undominated Nash Equilibrium Assignments for Example 2 Assuming Co-Existence[23]

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2, 4	3, 5, 6, 7, 8		1018	628

Table 16: Nash Equilibrium Assignment for Example 2 Assuming Pay to Connect Etiquette (Price of B = 1)

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2, 4	5, 6, 7, 8	3	1018	624

Table 17: Nash Equilibrium Assignment for Example 2 Assuming Simple Randomization Etiquette

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2, 4	3, 5, 6, 7, 8		1018	628

Table 18: Nash Equilibrium Assignment for Example 2 Assuming Informed Randomization Etiquette

Option A	Option B	Neither Option	Total surplus	Consumer surplus
4	1, 2, 3, 5, 6, 7, 8		609.593	479.593

[23] There are two Nash equilibria in total. Both equilibria achieve the same total and consumer surplus values as reported in this table.

Table 19: Undominated Nash Equilibrium Assignments for Example 2 Assuming Lump-Sum Willingness to Pay Etiquette[24]

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2	3, 4, 5, 6, 7, 8		857	309
1, 4	2, 3, 5, 6, 7, 8		857	309
1, 6	2, 3, 4, 5, 7, 8		857	309
1, 7	2, 3, 4, 5, 6, 8		857	309
2, 4	1, 3, 5, 6, 7, 8		857	309
2, 6	1, 3, 4, 5, 7, 8		857	309
2, 7	1, 3, 4, 5, 6, 8		857	309
4, 6	1, 2, 3, 5, 7, 8		857	309
4, 7	1, 2, 3, 5, 6, 8		857	309
6, 7	1, 2, 3, 4, 5, 8		857	309

[24] There are 32 equilibria in all. Table 13 shows the 10 equilibria involving weakly un-dominated strategies. In the full set of Nash equilibria, total surplus is equal to 857 in each equilibrium, and consumer surplus ranges between 207 and 387.