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Modeling the Efficiency of Spectrum Designated to Licensed Service and Unlicensed Operations

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Abstract

Obtaining the economically efficient use of spectrum requires that the price signals that guide its allocation and use be competitive. One approach involves establishing a non-exclusive licensing regime that provides for open and free access to spectrum. However, myopic, self-interested behavior on the part of users can give rise to excessive wireless spectrum congestion. This paper examines the Nash Equilibria that exist in an environment that exhibits characteristics present in the naturally occurring environment. The various Nash Equilibria generate total surplus levels that are far below the levels achieved under the efficient allocation of spectrum. This paper also examines the performance properties of congestion etiquettes that utilize various types of user information to address the congestion problem. Termed "market-informed" etiquettes, Nash Equilibrium theory predicts that these etiquettes are superior to the currently and widely used congestion etiquette, despite the fact that such etiquettes introduce a surprising strategic effect that may substantially reduce total welfare.

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1 Introduction

Recent advances in radio technology have made it possible to transmit data, voice, and video in digital form over high-speed wireless networks. In response to high demand, service providers and equipment manufacturers are constantly attempting to improve the transmission capacity of the available spectrum in an effort to avoid wireless congestion. For its part, the Federal Communications Commission (FCC) has the ability to reduce congestion by allocating additional spectrum for the provision of such services. To do so, the FCC must first determine whether to allocate spectrum to either licensed or unlicensed use. It is generally agreed that, to date, the allocation of spectrum to unlicensed use has significantly enhanced consumer welfare. However, the issue of whether to allocate additional spectrum to unlicensed use is both a contentious and complex one.

One complicating factor is that allocating additional spectrum to unlicensed use may, under certain conditions, reduce consumer welfare. The economic problem is straightforward. Markets work best when the choices that economic actors make reflect the cost that their decisions impose on society. Economic agents typically incur the cost of their decisions by paying an appropriately informed market price. Due to the free entry condition inherent in unlicensed use (as traditionally conceived), there is no assurance that consumers will take into account the negative effect of their spectrum consumption decisions on the value other consumers place on using the same spectrum. Thus, rational users may "over-consume" freely available spectrum compared to the level that would promote both consumers' and society's interests. This welfare reducing outcome is referred to as the "Tragedy of the Commons."

One solution involves treating spectrum allocated to unlicensed use as a "common pool resource" and utilizing a *congestion etiquette* to allocate spectrum to competing users. Most existing etiquettes are based solely on engineering principles, as opposed to market principles. This study defines four new potential congestion etiquettes, and examines the Nash equilibrium outcomes of each when spectrum users are allowed to choose between licensed and unlicensed options. We show that while each of the etiquettes improves the performance of the congestion-prone service, by reducing or eliminating the Tragedy of the Commons problem, some of the etiquettes also have an unanticipated secondary effect of distorting user choices between the two services. In many cases, the secondary distortion outweighs the primary beneficial effect, and aggregate total surplus is lower under an etiquette than under ordinary Nash equilibrium. These results demonstrate that modest changes to the spectrum assignment process can potentially increase the value that society receives from allocating spectrum to unlicensed use, but that great care must be taken in choosing a specific mechanism in which unlicensed spectrum is allocated.

Our results have implications for potential future public policies regarding the designation of spectrum to licensed use versus unlicensed operations through an auction mechanism. Some of these issues are addressed in a recently released working paper by the authors.[3] More broadly, this work is also relevant to the ongoing discussion of broadband infrastructure development and competition policy for local telecommunications. In particular, an understanding of, and potential resolution of the Tragedy of the Commons problem for usage of wireless spectrum will help policy makers evaluate the viability of a third wireless competitor (in addition to cable and wire line telecommunications providers) in the market for "last mile" broadband connections.

2 User Selection Between Licensed and Unlicensed Service Options

The Tragedy of the Commons refers to a situation in which myopic, self-interested behavior leads to the excessive use of a common pool resource. Because of this excessive use, the welfare of individual users is not maximized, and further, society fails to obtain the most efficient use of its scarce resources. In some situations, users of a common pool resource can avoid the Tragedy of the Commons problem by tacit coordination in their usage decisions. For example, bus riders in many cities voluntarily agree to wait in line for the next bus. In other cases (e.g. commercial fishing limits) both voluntary industry agreements and explicit governmental regulations have been used to address the tragedy of the commons problem. While such coordination has been successful in these instances, the radio spectrum market has several characteristics that make user coordination particularly difficult.

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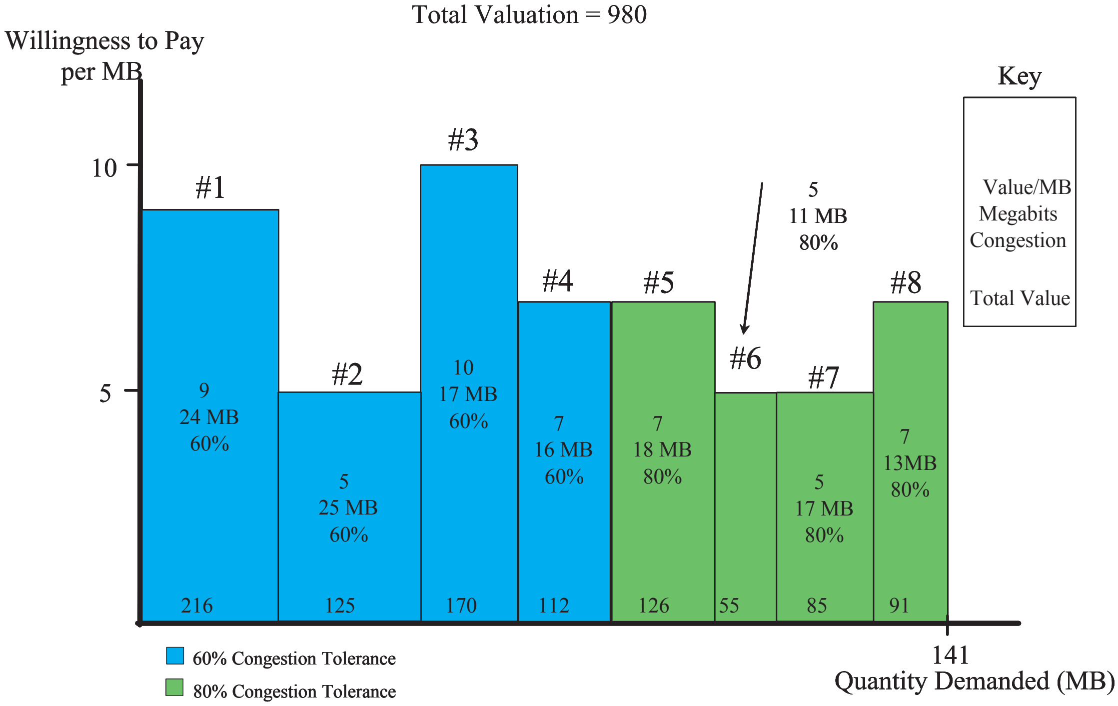
[3] Bykowsky, M., Olson, M., and Sharkey W. (2008) "A Market-based Approach to Establishing Licensing Rules: Licensed Versus Unlicensed Use of Spectrum," *OSP Working Paper #43*.

One market characteristic is the wide variation in uses to which users employ spectrum. Some users employ spectrum to download streaming video, while other users employ spectrum to send and receive text messages. The level of satisfaction a user obtains from employing spectrum depends importantly on the amount of unused capacity available to satisfy his or her desired application. Equally important, the level of available capacity needed to provide satisfactory streaming video service is different (in this case substantially greater) than the level needed to provide satisfactory text messaging service. Such differences in the amount of spectrum required by different applications complicate the ability of users to coordinate their uses. Therefore, in order to avoid the Tragedy of the Commons problem, users must not only recognize that their spectrum use negatively affects other users, but they must also take into account the specific demands of other users. As an example, the economic cost of adding one additional user who wishes to download streaming video may be negligible if all current users are employing spectrum for text messaging, but it may be very high if existing users are also downloading streaming video applications, or are otherwise intolerant of spectrum congestion.

A simple example can be used to describe the unique and complicated nature of the Tragedy of the Commons problem involving spectrum usage. Assume that there are eight different spectrum users, each of which is defined by a set of characteristics. Each user varies in the minimum amount of bandwidth that he or she needs in order to satisfy their service needs, as well as the value they place on having their service needs fully satisfied. Because of differences in service needs, users also vary in the extent to which they can tolerate spectrum congestion. For example, a user who requires a very high quality of service (e.g. video streaming or other applications that require a high and reliable data transmission rate) might find quality unacceptable whenever aggregate demand exceeds 60% of system capacity. Users who require a lower quality of service (e.g. email) would find quality acceptable if aggregate demand exceeds a higher percentage of system capacity (e.g. 80%). In what follows we measure a user's demand for service quality by his or her "congestion limit," expressed as the maximum amount of spectrum congestion the user can tolerate before the value he or she places on employing spectrum falls from some desired value to zero.[4] Figure 1 presents hypothetical data for a set of spectrum users, including the value that each user places on spectrum (expressed on a per megabit basis), the minimum amount of spectrum required by the user, and each user's quality of service requirement as measured by his/her congestion tolerance limit.

Figure 1. Spectrum User Characteristics in Example 1



Suppose that each user in the above example has the option to choose between a reliable and non-congestible transmission system that requires a fixed payment $P_A = 130$ (Option A), and a transmission system (Option B) that is "free," but where the value to each user depends on the system congestion level. The capacity of the free service is assumed to be equal to 100 units. The identification of the efficient assignment of spectrum depends on the amount of spectrum demanded by the users, the value that users

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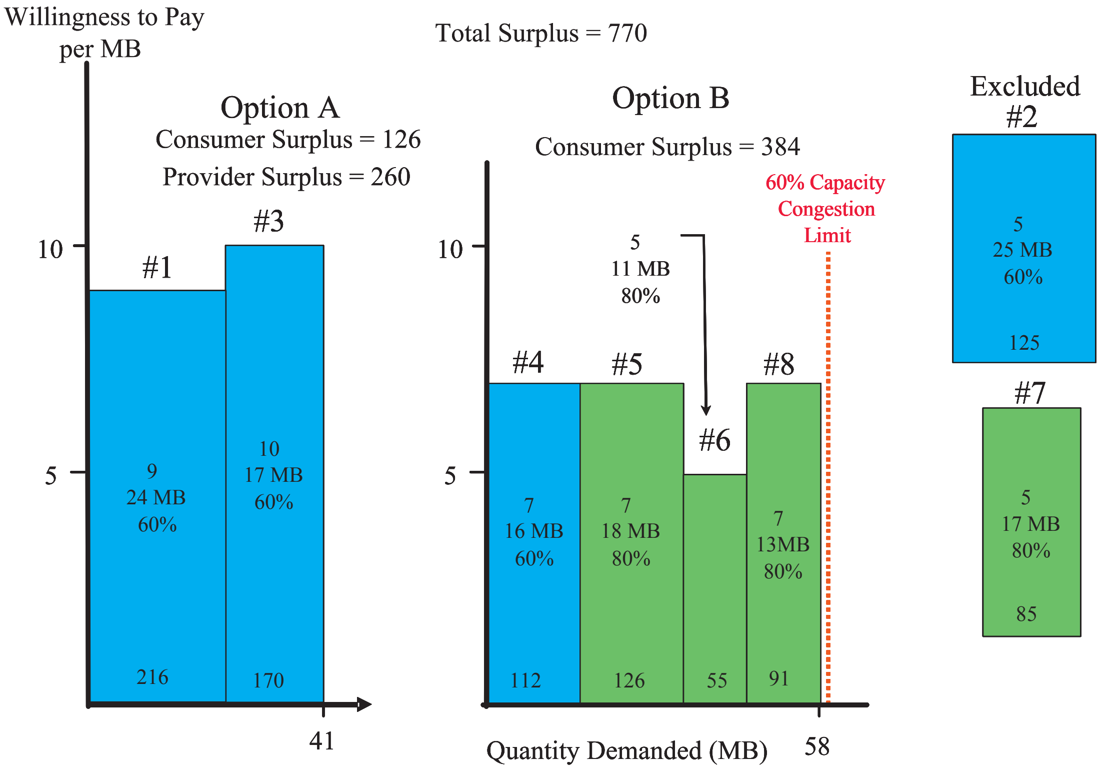
[4] A user's demand for quality is, in fact, a demand for a number of data transmission characteristics including file transfer rate, latency tolerance, and allowable level of jitter. These service attributes tend to degrade quite rapidly when total user demand exceeds a given percentage of total system capacity, which may vary by user. While an "all or nothing" congestion limit is an obvious simplification, it captures all of the strategic considerations of user selection, which is the main focus of this paper, and avoids the need to model the full complexity of choices between services with differing quality characteristics.

place on this spectrum, and the minimum quality of service that each user requires in order to obtain that value.

The unique efficient assignment of spectrum in this example involves assigning spectrum to users as shown in Figure 2. In Figure 2 and later tables, total surplus is defined as $\sum_{i \in A} v_i q_i + \sum_{i \in B} v_i q_i$, and consumer surplus is defined as $\sum_{i \in A} (v_i q_i - P_A) + \sum_{i \in B} (v_i q_i - P_B)$, where q_i represents user i's demand, v_i represents value per unit of demand for user i, and P_A and P_B represent the prices for options A and B respectively.[5]

Figure 2. Efficient Assignment of Spectrum in Example 1



Notice that in the efficient assignment, users 2 and 7 are excluded from spectrum available via both Options A and B. Under Option A, both users 2 and 7 would receive

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[5] By default, the price of Option B is equal to zero. Under some etiquettes, to be discussed later, a positive price might be assigned. In the following section, a per unit price, P_B, may be assigned to users of Option B. In the above notation, producer surplus is defined as $P_A\ n_A + P_B\ n_B$, where n_A and n_B represent the number of subscribers choosing options A and B respectively.

negative surplus, since the value they place on service is less than the subscription fee of 130. Moreover, including either user 2 or user 7 under the free service (Option B) would require displacing at least one of the existing users of that service, and one can verify that any such change would result in a reduction in total surplus.

2.1 Tragedy of the Commons Problem: Nash Prediction

Suppose users simultaneously and independently choose whether to select the subscription service (Option A), the free service (Option B), or neither service. A Nash equilibrium is a particular joint selection such that no individual user could increase his or her payoff by selecting an alternative strategy, assuming that all other users maintain their equilibrium strategy choices. We can now see immediately that the efficient assignment in Example 1 is not a Nash equilibrium. Suppose that user 7 contemplates joining the free service. Such a change would increase the total demand for this service from 58 to 75 units, but since user 7 expects to receive acceptable quality of service as long as total demand is less than 80 units, this alternative selection would increase user 7's surplus from zero to 85 valuation units.[6] Under Nash equilibrium assumptions, user 7 would therefore choose this alternative. User 7's deviation from the efficient allocation unfortunately reduces total surplus. With a total demand of 75 for Option B, user 4 no longer receives a satisfactory quality of service, and under our assumptions, her entire surplus of 112 valuation units is assumed to be forfeited.

While the efficient assignment cannot be sustained as a Nash equilibrium, there are alternative joint user selections that satisfy the Nash equilibrium conditions. The magnitude of the Tragedy of the Commons problem in Example 1 is measured by the difference between the total surplus under the efficient assignment and total surplus under the Nash equilibrium condition. The value of this measure, as predicted by Nash Equilibrium Theory, depends on the joint user selection. One common element of each Nash assignment (in this example) is that both users 1 and 3 are assigned to the subscription service (Option A). To see this, suppose that users 1 and 3 select Option A, users 5, 6, 7, and 8 select Option B, and that users 2 and 4 choose neither option. These

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[6] User 7 has a congestion tolerance of 80%. Given that the free service's capacity is 100 Mb, user 7 would obtain full value from selecting Option B

selections satisfy the Nash equilibrium conditions. If either user 1 or user 3 attempts to switch to the free service in order to avoid paying the subscription fee, demand for Option B would rise above either of their congestion limits, and they would forfeit the value received by selecting Option A. Each of the users choosing Option B would clearly see a reduction in surplus by switching to Option A, since they would be required to pay the subscription fee of 130, under which they receive negative surplus. Finally, users 2 and 4 receive zero surplus by choosing neither option, but would do no better by choosing either Option A or Option B.[7]

Table 1. Nash Equilibrium Assignments for Example 1

User Choice Selection			Performance Measures		
Option A	**Option B**	**Neither Option**	**Total Surplus**	**Consumer Surplus**	**Total Efficiency**
1, 3	*5, 6, 7, 8*	*2, 4*	*743*	*483*	*96.5%*
1, 3	*4, 5, 6, 7, 8*	*2*	*743*	*483*	*96.5%*
1, 3	*2, 6, 7, 8*	*4, 5*	*617*	*357*	*80.1%*
1, 3	*2, 5, 7, 8*	*4, 6*	*688*	*428*	*89.4%*
1, 3	*2, 5, 6, 8*	*4, 7*	*658*	*398*	*85.5%*
1, 3	*2, 5, 6, 7*	*4, 8*	*652*	*392*	*84.7%*
1, 3	*2, 5, 6, 7, 8*	*4*	*386*	*126*	*50.1%*
1, 3	*2, 4, 7, 8*	*5, 6*	*562*	*302*	*73.0%*
1, 3	*2, 4, 6, 8*	*5, 7*	*532*	*272*	*69.1%*
1, 3	*2, 4, 6, 7*	*5, 8*	*526*	*266*	*68.3%*
1, 3	*2, 4, 6, 7, 8*	*5*	*386*	*126*	*50.1%*
1, 3	*2, 4, 5, 8*	*6, 7*	*603*	*343*	*78.3%*
1, 3	*2, 4, 5, 7*	*6, 8*	*597*	*337*	*77.5%*
1, 3	*2, 4, 5, 7, 8*	*6*	*386*	*126*	*50.1%*
1, 3	*2, 4, 5, 6*	*7, 8*	*567*	*307*	*73.6%*
1, 3	*2, 4, 5, 6, 8*	*7*	*386*	*126*	*50.1%*
1, 3	*2, 4, 5, 6, 7*	*8*	*386*	*126*	*50.1%*
1, 3	**2, 4, 5, 6, 7, 8**		**386**	**126**	**50.1%**

Total surplus in the above equilibrium selection is equal to 743 valuation units. In fact, this is the highest total surplus obtainable in any of the Nash equilibria, although it is lower than the surplus of 770 obtained under the efficient assignment. Thus, if this equilibrium was obtained, the social cost of the Tragedy of the Commons problem would

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[7] All of the assertions made about specific Nash equilibrium outcomes can be verified by simple computations. Results presented in later tables are based on computer algorithms which are able to exhaustively repeat these computations for all possible joint strategies.

be very small. However, as noted earlier, there are many joint user selections that satisfy the Nash equilibrium conditions, many of which generate much lower levels of total surplus. The full set of Nash equilibria and the associated total efficiency levels for Example 1 are shown in Table 1.[8]

While there a large number of outcomes predicted under the assumption that users follow Nash equilibrium behavior, there are strong theoretical grounds for assuming that the outcomes that generate the greatest amount of social loss are the ones that are the most likely to occur. To see this, note that users who select Option B are guaranteed to receive at least a zero payoff, and there is a possibility that they could receive a positive payoff if other users deviate (contrary to Nash assumptions) from the assumed equilibrium. Those who select "neither option" receive a zero payoff with certainty. In the language of non-cooperative game theory, in this case a strategy in which a user chooses "neither option" is "weakly dominated" by the strategy in which that user chooses Option B. The only equilibrium in which no user selects a weakly dominated strategy is shown in the last row of Table 1. This outcome is the worst of all Nash equilibrium outcomes, measured in terms of both total and consumers' surplus, and its total efficiency is equal to 50.1%.

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[8] In some cases all of the users who select Option B receive zero surplus due to the congestion caused by their collective choices. However, these outcomes satisfy the Nash equilibrium conditions since no individual user can improve his or her payoff by selecting an alternative strategy. For example, when users 1 and 3 select Option A, users 2, 5, 6, and 7 choose Option B, and user 4 chooses neither option, all users except 1 and 3 receive zero surplus.

2.2 Alternative Congestion Etiquettes

In this section we consider a number of service etiquettes that can potentially lead to superior market outcomes. We first consider an etiquette that addresses the congestion problem by changing the payoffs of users and, in so doing, their service choice. This etiquette is based on the observation that the Tragedy of the Commons problem is made worse by the fact that the strategy of selecting Option B weakly dominates the strategy of selecting neither Option A nor B. One method of changing user incentives, herein called the "Pay to Connect" etiquette, is to charge users a small non-refundable fee if they select Option B. Under pay to connect, a user who selects Option B, but who also expects that his/her decision would raise the demand for spectrum available under Option B to a level above his or her own congestion limit, would expect to receive negative surplus, and would therefore prefer to choose to sit on the sidelines and earn a zero payoff.

Given a fee for selecting Option B, the last row of Table 1 would no longer represent a Nash equilibrium. Similarly, *any* equilibrium from Table 1 in which at least one user receives zero surplus while choosing Option B cannot be an equilibrium under the Pay to Connect option. In fact, in Example 1, the only Nash equilibrium strategy selection that survives the imposition of a small but positive fee for Option B is the highest surplus outcome in Table 1. In this outcome total efficiency is equal to 96.5%.

We next consider two etiquettes that employ a rationing technique to more efficiently allocate spectrum to those users that select Option B. As before, users first decide whether to select Option A or Option B. For those who select Option B, a random number generator is used to assign a priority level to each such user. If *n* users select Option B, then there are *n!* permutations of users, and each permutation is assumed to be chosen with equal probability.[9] Users are then assigned spectrum under Option B until the total demand exceeds a pre-determined level, which we assume to be equal to the highest congestion limit of all users who are expected to make use of the system.[10] Herein

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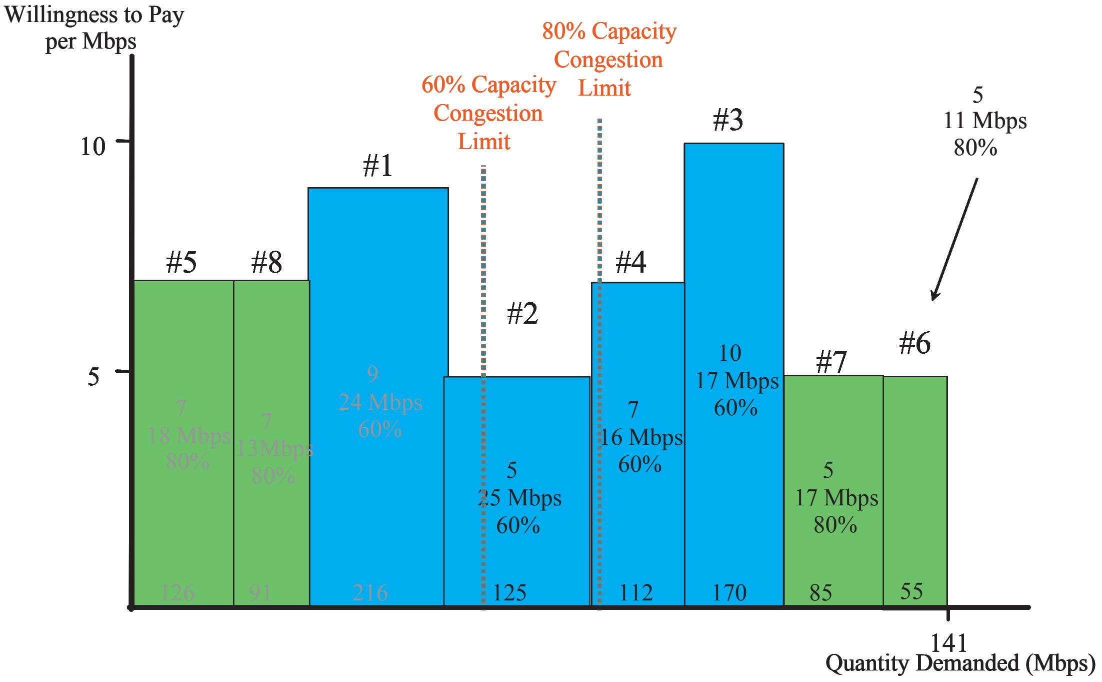
[9] When all 8 users choose Option B, there are 40,320 possible permutations.

[10] In an alternative version of this randomization etiquette, users could be served only until total demand exceeds the lowest congestion limit of expected users. There are clear trade-offs involved in selecting the appropriate system capacity, and neither rule is optimal in all situations. With a high limit, some high priority users with low congestion limits would be granted service under the etiquette, but the service would be worthless to them if the total demand exceeded their personal congestion limit. On the other hand, there can be situations in which some users would be served when system capacity is set at the high congestion limit but would be rejected under the lower limit.

referred to as "Simple Randomization," this etiquette addresses the tragedy of the commons problem by guaranteeing that some users (those with high priority and high congestion limits) receive some value from their assigned spectrum under Option B. A particular implementation of the simple randomization etiquette for Example 1 is illustrated in Figure 3 below. In this example, all 8 users are assumed to select Option B, and the random priority assignment ranks users in the order {5, 8, 1, 2, 4, 3, 7, and 6}.

Figure 3. Simple Randomization in Example 1



Under the simple randomization etiquette, users 5, 8, 1, and 2 are provided spectrum, while all lower priority users are excluded. Including any additional user would raise total demand above the system capacity of 80, which is equal to the maximum congestion limit. In this case, users 1 and 2 would receive no value from their spectrum assignment, since the total demand would exceed their personal congestion limits, and the resulting total efficiency would be equal to 16.3%. Alternative user selections between Options A and B will result in different outcomes under the Simple Randomization etiquette. When users take all such random outcomes into account there is a unique Nash equilibrium whose outcome is shown in Appendix A. In this equilibrium, total efficiency is equal to 83.6%.

In an alternative version of the randomization etiquette, to be called "Informed Randomization", users are first asked to report their personal congestion limits to a system manager.[11] As in simple randomization, a random number generator is used to assign a priority level to each user who selects Option B. Service is now assigned, however, by taking account of the congestion limits of each potential user. If the demand of the highest priority user is less than that user's congestion limit, that user is guaranteed service. The etiquette then considers each additional user's demand and personal congestion limit in order of decreasing priority. The etiquette ensures that total demand always remains less than or equal to the minimum congestion limit of all users who are guaranteed service. With each new user, the relevant system congestion limit is set equal to the minimum limit of the current user being considered, and the limits of all higher ranking users already guaranteed service. If the total demand, including the current user, is less than or equal to the congestion limit, that user is granted service. Otherwise, that user is not served and the next user is considered. The etiquette continues to examine users until all have been considered, or until demand is exactly equal to the system limit as determined by the currently served users.

Under the Informed Randomization Etiquette, users 2, 4, 5, 6, 7, and 8 would expect to receive positive surplus under Option B, since there is positive probability that they will be assigned a high priority rank. They will therefore choose Option B in preference to choosing neither Option A nor Option B. In addition, user 3 in this example has an incentive to select Option B over Option A, because the *expected* surplus to him or her under the randomization protocol is higher than could be achieved *with certainty* by

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[11] The system manager would most likely be embodied in a hardware devise, and user reports about personal congestion limits would correspond to priority bits under existing internet protocols. The sensitivity of the randomization (and later) etiquettes to personal congestion limits raises the question of whether users will have the incentive to report their limits truthfully. In fact, "truth telling" is in this case is a weakly dominant strategy (in game theoretic terminology) since a user who selects Option B can never increase his or her payoff by falsely reporting a congestion limit. To see this, note that a high priority user with a low (e.g. 60%) congestion limit would never wish to report a higher limit. If the user would have been excluded under a truthful (60%) report, than, while a false report (e.g. 80%) might result in providing service, the service would be of no value to the user. Similarly, a user with a high congestion limit would never wish to report a lower limit, since in some cases this would result in that user being denied service, while in no cases would a lower total demand increase the quality of service under the present assumptions.

choosing Option A. For user 1, the certain surplus under Option A is higher than that under the randomization etiquette, whether or not user 3 chooses Option B. The resulting total efficiency is equal to 72.8%.

While both versions of the randomization etiquette guarantee that those users who select Option B, and who have a sufficiently high priority level, receive positive surplus, these protocols do not necessarily create the correct incentives for users to make optimal selections between Options A and B. In particular, by raising a user's expected return from selecting Option B, the randomization etiquettes may induce high valued users (who can afford to select Option A) to select Option B. While this choice is privately welfare maximizing for the user, it can potentially reduce total surplus.

Finally, we consider an etiquette that utilizes reported information on a user's congestion tolerance and willingness to pay (WTP) to more efficiently allocate spectrum to those users who select Option B. In one version of a WTP etiquette (hereafter called the "Lump Sum WTP Etiquette"), users who select Option B would be asked to report both their personal congestion limit and their willingness to pay to receive or send information. The reported willingness to pay values would then be used to define a priority ranking of users, and the etiquette would proceed in a similar manner as the randomization etiquette (but without the random component). In one important difference, however, users under the WTP etiquette would be required to actually pay a price to receive service. Under this etiquette, the price is determined by the reported willingness to pay of the marginal user who is excluded from service by the etiquette. Only users who are guaranteed service under the etiquette are required to pay this price.[12]

As an example, suppose that in Example 1, all users bid "truthfully" in the following sense: users 1 and 3 bid 130, which is the price they would have paid if they had selected Option A, and all remaining users bid their actual value, which in each case is less than 130.[13] Users 1 and 3 would then be tied for the highest priority of service, and they would be followed in order by users 5, 2, 4, 8, 7 and 6. Under the WTP etiquette, users 1,

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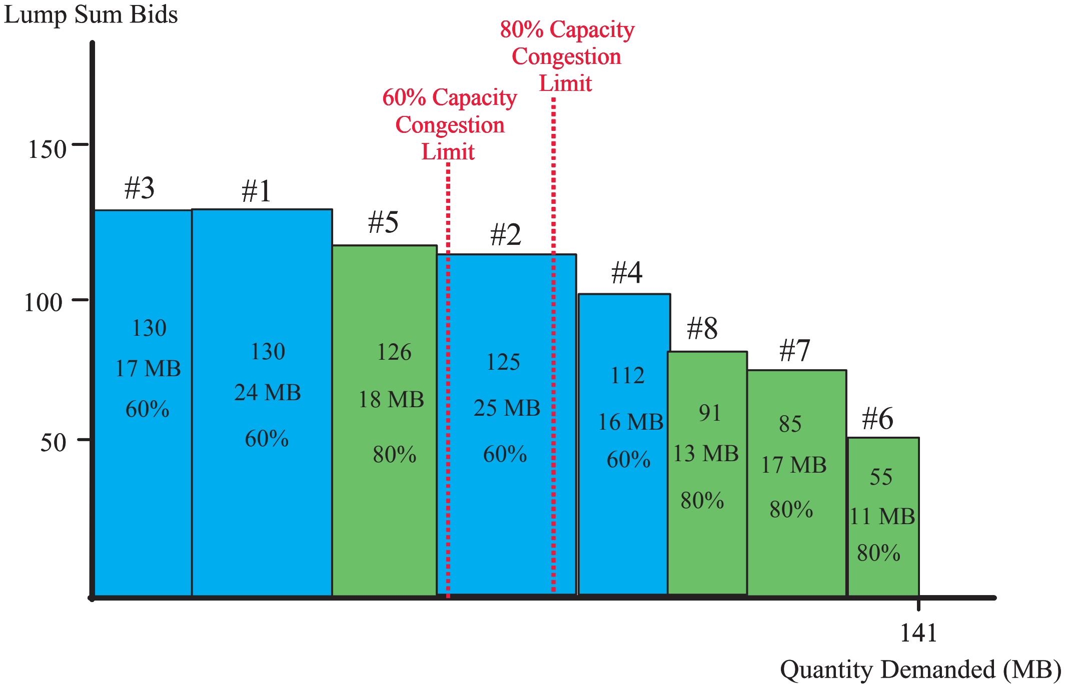
[12] This pricing rule is based on standard "second price" auction theory. In the present context, there is some ambiguity about the definition of the marginal user. The present version of the etiquette assumes that the extra-marginal user is defined as the highest priority user who is excluded from service, and such that no lower priority user is assigned service.

[13] In general, truthful bidding of this form is not guaranteed. While truthfully revealing a personal congestion limit is still a weakly dominant strategy, there can be situations in which users may prefer to misrepresent their willingness to pay in an effort to lower the market price.

3 and 5 would be assigned spectrum under Option B, and each would pay a price equal to 125, which is the highest rejected bid. No other users will be assigned spectrum, since the addition of any other user would violate the requirement that total demand remain less than or equal to the minimum congestion limit of all users who are guaranteed service.

Figure 4: Willingness to Pay in Example 1



The ability of users to express their willingness to pay introduces some surprising and important strategic effects. For example, users with high valuations can now predict with greater certainty whether or not they will be served if they choose Option B. No user who can afford to pay the subscription price for Option A can gain by choosing Option B and making a bid higher than the subscription price for Option A (for which satisfactory service is guaranteed).[14] Assuming that users place "truthful" bids as described above, all users select Option B in the set of undominated Nash equilibria for Example 1. In this equilibrium both total surplus and consumer surplus are lower than surplus under both of the randomization etiquettes and under and Pay to Connect. Total efficiency under the WTP etiquette for Example 1 is equal to 66.5%.

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[14] A bid for service under Option B that is higher than the price of Option A would differ from a bid equal to the price of Option A only if the bid of the marginal user under Option A was higher than the price of Option A. In such a case, the bidder would be better off choosing Option A instead of Option B.

2.3 Summary Analysis of Alternative Congestion Etiquettes[15]

In this section we will summarize the above analysis as it applies to Example 1, as well as three additional examples, which are reported in detail in Appendix A. In each example, there are eight different users. Demand for bandwidth, the value of completed service, and the demand for service quality as defined by a personal congestion limit varies among the users. The examples differ primarily in terms of the set of users who can potentially afford to purchase the subscription service under Option A. In Example 1, with a price for Option A equal to 130, two out of eight users can afford Option A as illustrated in Figure 1. In Example 2, the price of option A remains equal to 130, but user demands and values of service are changed in such a way that five out of eight users can afford Option A. Examples 3 and 4 maintain the demand and value of service assumptions of Example 2, but adjust the price of Option A with the result that eight out of eight users can afford Option A in Example 3, while zero out of 8 can afford Option A in Example 4.

The examined etiquettes fall into two categories. One class of etiquette attempts to solve the congestion problem by changing the payoffs that some users receive by selecting Option B. In the Pay to Connect etiquette, users pay a small fee for selecting Option B, and have an incentive to refrain from selecting this option when it is expected that their decision would lead to an outcome that is both individually non-rational and globally inefficient. Thus, the Pay to Connect option can lead to superior choices between Options A and B, but at the same time, it does not lead to efficient spectrum allocation among the users who finally choose Option B.[16]

Another class of etiquettes attempts to more efficiently allocate spectrum among those users that choose Option B. Both the Simple and Informed Randomization etiquettes work by explicitly denying service to low priority users when total demand threatens to exceed a pre-defined system congestion limit. However, randomization has a potentially perverse effect on user choices between Options A and B. On the one hand, it gives every user who cannot afford to choose the subscription service (Option A) a

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[15] Data for three additional examples are provided in an Appendix.

[16] In general, Pay to Connect does not result in a unique equilibrium, or guarantee that the highest equilibrium surplus is attained.

positive incentive to choose Option B, rather than choose neither option. It can also give some users an incentive to switch from Option A to Option B, in cases where their expected payoff in Option B is higher than the certain payoff in Option A. The latter effect is more pronounced under the informed version of the etiquette than under the simple version. As with the Pay to Connect etiquette, an etiquette based on the randomized ranking of users does not ensure that the spectrum available under Option B is assigned to those users that value such spectrum the most.

The Willingness to Pay etiquette has both potential advantages and potential disadvantages compared to the randomization etiquettes. User willingness to pay can potentially allocate service to the highest value users who choose Option B. However, high value users (users who would be willing to pay the subscription price for Option A) can reduce the uncertainty regarding whether they will obtain service via Option B by submitting a bid that is equal to the cost of accessing spectrum via Option A. As a result of this reduction in uncertainty, more high value users now have an incentive to select Option B, and these decisions can potentially reduce total surplus by generating more potential congestion under option B.

The results of all four examples, presented in detail in Appendix A, are summarized in Tables 2 and 3 below. In these tables, efficiency is defined as the surplus obtained under the corresponding etiquette, as a fraction of maximum possible surplus, expressed in percentage terms. Each row reports only the equilibrium surplus associated with un-dominated strategies under the corresponding etiquette. Examples 3 and 4 were deliberately chosen as extreme cases, where the co-existence equilibrium achieves full efficiency in Example 3 and zero efficiency in Example 4. These results are explained as follows. In Example 3, all users can afford to purchase Option A, so that in equilibrium, no user would willingly choose service under Option B while expecting that enough other users would do so to raise total demand above his or her personal congestion limit. Since Option A is available in this case, that option dominates a choice of Option B. In Example 4, the situation is reversed. No users can afford to purchase Option A, and therefore, in an undominated strategy equilibrium, all users must choose Option B. Any user for whom total demand for Option B divided by total system capacity for Option B is greater than his or her personal congestion limit can do no better than a zero payoff in this case. In Example 4, all users happen to fall into this category under the assumed parameters.

In each of the examples, two of the four pro-active etiquettes (pay to connect and simple randomization) achieve higher total surplus than that achieved under simple co-existence.[17] Informed randomization achieves higher total surplus in only two of the four examples, while the willingness to pay etiquette achieves equal or higher surplus in three of the four examples. In terms of consumer surplus, the results are similar. Simple randomization achieves outcomes that are equal to or superior to the co-existence outcome. Pay to connect achieves higher consumer surplus in two of the four examples and marginally smaller surplus in the remaining two cases. The informed randomization and willingness to pay etiquettes achieve higher consumer surplus in two of the four examples and lower surplus in the remaining two cases.

Table 2. Summary of Total Surplus Efficiency Results

Etiquette	Example 1	Example 2	Example 3	Example 4
Co-existence	50.1%	91.4%	100%	0%
Pay to Connect	96.5%	91.4%	100%	84.8%
Simple Randomization	83.6%	91.4%	100%	43.6%
Informed Randomization	72.8%	54.7%	93.1%[18]	75.0%
WTP	66.5%	76.9%	100%	100%

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[17] Note, however, that some co-existence equilibria involving dominated strategies (where one or more users select neither option and receive a zero payoff with certainty) can achieve higher total surplus than some of these etiquettes.

[18] This result and the corresponding entry in the next table represent an average of gross surplus due to four equilibria as presented in Appendix A.

Table 3. Summary of Consumers' Surplus Efficiency Results

Etiquette	Example 1	Example 2	Example 3	Example 4
Co-existence	23.9%	100%	100%	0%
Pay to Connect	90.7%	99.4%	99.6%	84.1%
Simple Randomization	72.6%	100%	100%	43.6%
Informed Randomization	81.5%	76.4%	91.5%	75.0%
WTP	25.9%	49.2%	78.1%	18.7%

3 A Simple Competitive Model

In this section we introduce a simple supply side to the analysis in order to provide further insights into the question of spectrum allocation between licensed and unlicensed uses. In contrast to previous sections, where the subscription service (Option A) was assumed to be offered at a fixed price, and a second service (Option B) was assumed to be offered at fixed capacity (either on a free basis or with consumer fees based on certain congestion etiquettes), this section will consider the possible market behavior in cases in which spectrum, like many other production inputs, is freely available at a constant unit cost. While this assumption is unrealistic, given the way that spectrum is currently allocated, it may provide some insights into the sources of market behavior (and possible market failure) that may underlie spectrum allocation which this paper is concerned with. Throughout this section, we allow both the capacity of Option B and the assumed pricing behavior by a competitive supplier of that option to vary.

In order to model the supply side, we now assume that service providers for Options A and B are required to acquire spectrum at a constant unit cost equal to m per megabit. Under Option A, the service provider is assumed to provide guaranteed access of k units of capacity to each subscriber at a lump sum price P_A. Under Option B, the service provider is assumed to provide K units of total capacity, at a cost of $m K$, that can be sold to users at a per-unit price $p_B \geq m$. Subscription service under Option A is assumed to be provided by a single monopoly supplier, who may be able to earn positive profits (i.e. P_A

$> m\ k$).[19] In contrast, Option B is assumed to be competitively supplied, so that the price $p_B = \frac{mK}{\sum_{i \in B} q_i}$ satisfies a zero profit constraint. Note that, in general, $p_B > m$ since K must be larger than the total demand of users who select Option B in order to satisfy their congestion limits.

As in the previous section, we continue to assume that spectrum users individually and simultaneously decide whether to subscribe to service under Option A (with lump sum price P_A) or to use spectrum provided by a supplier of Option B (at an endogenously determined per unit price p_B), or to choose neither option. The subscription price P_A is assumed to be independent of the number of users who select that option, and service under this option is assumed to be congestion free. In contrast, the per-unit price p_B of Option B is assumed to depend on the set of users who ultimately select that option, and the capacity of Option B is assumed to depend on the service providers expectation about those users expected to select it.[20] Under these assumptions, both optimal and Nash equilibrium outcomes can be computed in exactly the same manner as in the previous section. As above, we can summarize these outcomes in terms of gross and net efficiency, which measure respectively the percentage of total and consumers' surplus as a percentage of the maximum corresponding surplus.

Table 4 presents a detailed description of the possible Nash equilibrium outcomes that might occur under the competitive scenario under the assumptions of Example 1.[21] In that example, the lump sum price of Option B is equal to 130, and the service demands and values are described in Figure 1. Now, however, the capacity of Option B is assumed to be variable, and as described above, users who select that option are assumed

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[19] Given our previous assumption that service under Option A is congestion free, we also assume (and all of our examples will verify) that the price P is sufficient to recover the capacity cost ($m\ q_i$) of each subscriber of Option A, or alternatively that the aggregate revenues of all subscribers are sufficient to recover total capacity costs. In addition, we assume that each subscriber to Option A is guaranteed a dedicated capacity of service, so that congestion costs are not relevant to any user who selects that option.

[20] This is a subtle point, which may become more clear after the detailed discussion of Example 1 is presented. Essentially, our analysis assumes that capacity of Option B is consciously chosen in order to maximize total gross surplus, which is in turn based on equilibrium user selections given any particular level of capacity. Since there may be multiple equilibria associated with any given capacity choice, there can be no guarantee that the optimal gross surplus is attained in every possible Nash equilibrium, although we will demonstrate that for each "optimal" capacity level, at least one equilibrium must attain this level of total surplus.

[21] None of the equilibria in Table 4 involve the use of weakly dominated strategies.

to be charged a per-unit price which guarantees that the service provider of that option will earn zero economic profits. In Table 4, we assume that the marginal cost of spectrum capacity is equal to 1.

Table 4: Nash Equilibria in Example 1 with Variable Capacity and a Zero Profit Constraint for Option B with Marginal Cost = 1

Parameters		Equilibrium User Selections			Performance	
Capacity Option B	Price Option B	Option A	Option B	Neither Option	Total Surplus	Consumer Surplus
100	*1.69*	*1,3*	*4,5,6,7,8*	*2,4*	*743*	*383*
166.667		*1,3*		*2,4,5,6,7,8*	*386*	*126*
166.667	*1.81*	*1*	*3,4,5,6,7,8*	*2*	*855*	*558.333*
166.667	**1.67**	**1,3**	**2,4,5,6,7,8**		**980**	**553.333**
166.667	*1.68*	*3*	*1,4,5,6,7,8*	*2*	*855*	*558.333*
166.667	*1.67*		*1,3,5,6,7,8*	*2,4*	*743*	*576.333*
195		*1,3*		*2,4,5,6,7,8*	*386*	*126*
195	**1.67**	**1**	**2,3,4,5,6,7,8**		**980**	**655**
195	*1.68*		*1,3,4,5,6,7,8*	*2*	*855*	*660*
195	*1.81*	*3*	*1,2,5,6,7,8*	*4*	*868*	*543*
206.667		*1,3*		*2,4,5,6,7,8*	*386*	*126*
206.667	**1.77**	**1**	**2,3,4,5,6,7,8**		**980**	**643.333**
206.667	*1.78*		*1,3,4,5,6,7,8*	*2*	*855*	*648.333*
206.667	**1.67**	**3**	**1,2,4,5,6,7,8**		**980**	**643.333**
235		*1,3*		*2,4,5,6,7,8*	*386*	*126*
235	**1.67**		**1,2,3,4,5,6,7,8**		**980**	**745**

The first row of the table assumes that capacity of Option B is equal to 100, which is the same assumption maintained in the previous section. The results from this row are therefore directly comparable to the results for the various congestion etiquettes described above. These results show that when a per-unit price for Option B is set in order to satisfy a zero profit constraint for Option B, the resulting total surplus corresponds exactly to total surplus attained under the Pay to Connect etiquette. These surplus levels are generally, though not universally, higher than total surplus resulting form the other etiquettes. For consumer surplus, the results lie between those attained by the congestion etiquettes.[22]

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[22] Recall, however, that these etiquettes do not account for any costs associated with provision of Option B.

Rows 2 through 15 of Table 4 assume that capacity of Option B is chosen in order to maximize total surplus. One can verify that when marginal cost of spectrum capacity is equal to 1, the maximum possible total surplus in this example is equal to 980, and an inspection of the total surplus column in the table shows that this level of surplus is attained at four different levels of capacity. For each level of capacity, however, there are multiple equilibria based on user choices, and so the maximum possible surplus cannot be guaranteed without further coordination. While the maximum total surplus is equal to 980, there are Nash equilibria in which total surplus can be as low as 386. These inefficient equilibria occur because the capacity for Option B is assumed to be chosen under the assumption that a particular set of users will ultimately choose that option. However, under the current pricing assumption, there can be additional equilibria in which fewer, or even none, of these users can (in equilibrium) rationally choose Option B given the expected choices of other users. These inefficient equilibria occur in part because of our maintained assumption that prices are set after user selection.

For consumer surplus, a similar analysis is possible. The last row of Table 4 describes the outcome when capacity of Option B is set equal to 235, and all users in equilibrium select that option. Both total surplus and consumer surplus attain their maximum values in this equilibrium. At this capacity level, however, there remains an alternative, and much inferior equilibrium, in which two users select Option A and the remaining users choose neither option.

Tables 5 and 6 present the results corresponding to a fixed capacity for Option B equal to 100 for each of the four examples considered above, and for varying levels of marginal cost. In both tables, each cell reports both the maximum possible surplus attainable and the best possible surplus attained in equilibrium, both in total and as a percentage of the maximum. A comparison of Tables 2 and 3 with Tables 5 and 6 shows that when marginal cost is equal to 2 or less, the gross efficiency results exactly match the results for the pay to connect etiquette. More generally, both the total surplus and consumer surplus results in Tables 5 and 6 closely follow the corresponding results of the pay to connect etiquette in Tables 2 and 3.

Table 5. Maximum Total Surplus and Efficiency Results in the Competitive Model with K = 100

		Example 1	Example 2	Example 3	Example 4
$MC_B=1$	Max TS	770	1114	1114	561
	Eq TS	743 (96.5%)	1018 (91.4%)	1114 (100%)	476 (84.8%)
$MC_B=2$	Max TS	770	1114	1114	561
	Eq TS	743 (96.5%)	1018 (91.4%)	1114 (100%)	476 (84.8%)
$MC_B=4$	Max TS	624	1018	1114	561
	Eq TS	386 (61.9%)	857 (84.2%)	1114 (100%)	0 (0%)

Table 6. Maximum Consumer Surplus and Efficiency Results in the Competitive Model with K = 100

		Example 1	Example 2	Example 3	Example 4
$MC_B=1$	Max CS	428	528	814	461
	Eq CS	383 (89.5%)	528 (100%)	814 (100%)	376 (81.6%)
$MC_B=2$	Max CS	328	428	714	361
	Eq CS	283 (86.3%)	428 (100%)	714 (100%)	276 (76.5%)
$MC_B=4$	Max CS	128	228	714	161
	Eq CS	126 (98.4%)	207 (90.8%)	714 (100%)	0 (0%)

As Table 4 illustrates, there are a large number of Nash equilibria possible in the competitive model, both because different levels of system capacity for Option B might be chosen, and because multiple equilbria for any given level of capacity are possible. For any given level of capacity, however, some of the equilibria require some users to refrain from choosing either Option A or Option B. As long as it is reasonable to assume that users who select Option B and are subsequently denied satisfactory service do not pay the competitive price for Option B, these equilibria are weakly dominated for exactly the same reason as discussed in the previous section.[23] If all weakly dominated equilibria are removed, only the five rows in Table 4 highlighted in bold remain as Nash equilibria.

Table 7 presents summary results for Example 1 and each of the remaining examples for the weakly undominated equilibrium corresponding to the highest capacity choice that maximizes consumer surplus.

Table 7: Nash Equilibrium Outcomes in the Competitive Model with Optimal Capacity for Option B

Examples	**Parameters**			**Equilibrium User Selections**[24]	**Performance**	
	P^A	K_B	p^B		**Total Surplus**	**Consumer Surplus**
Ex. 1, MC=1	130	235	1.67	{2,2,2,2,2,2,2,2}	980	745
Ex. 1, MC=2	130	235	3.33	{2,2,2,2,2,2,2,2}	980	510
Ex. 1, MC=4	130	73.8	5.0	{1,0,1,0,2,2,2,2}	743	188
Ex. 2, MC=1	130	230	1.67	{2,2,2,2,2,2,2,2}	1114	884
Ex. 2, MC=2	130	230	3.33	{2,2,2,2,2,2,2,2}	1114	654
Ex. 2, MC=4	130	80	5.0	{1,1,0,1,2,2,2,2}	1018	308
Ex. 3, MC=1	50	230	1.67	{2,2,2,2,2,2,2,2}	1114	884
Ex. 3, MC=2	50	80	2.5	{1,1,1,1,2,2,2,2}	1114	754
Ex. 3, MC=4	50[25]	0		{1,1,1,1,1,1,1,1}	1114	714
Ex. 4, MC=1	250	230	1.67	{2,2,2,2,2,2,2,2}	1114	884
Ex. 4, MC=2	250	230	3.33	{2,2,2,2,2,2,2,2}	1114	654
Ex. 4, MC=4	250	180	6.67	{2,2,0,2,2,2,2,0}	948	228

A comparison of Tables 5, 6 and 7 allows us to quantify the impact of adjusting capacity for Option B to optimal levels, rather than maintaining it at an arbitrary level of 100 units. In Example 1, with marginal cost is equal to 1, the highest possible total surplus with capacity for Option B equal to 100 is equal to 770. Table 7 reveals that both the equilibrium surplus and the maximum total surplus increase to 980 when capacity is

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[23] Rather than choosing neither option, and accept a zero payoff with certainty, any user would weakly prefer to choose Option B even if in equilibrium such a user would expect to receive a zero payoff.

[24] In this column the optimal strategy is represented by a vector describing the equilibrium strategy choice of each user, where 0 corresponds to neither option, 1 corresponds to Option A and 2 corresponds to Option B.

[25] Note that when marginal cost for Option B (and by assumption also for Option A) is equal to 4, high demand users who select Option A can only be provided their necessary capacity if a sufficient number of lower demand customers also subscribe to this option. For example, the cost of serving user 2 is equal to 100 while in this example, the lump sum price of service is equal to 50. In the equilibrium shown, the service provider for Option A is nevertheless guaranteed to earn positive profits.

set optimally at 235 units instead of 100. This represents a gain of 27% over the maximum surplus at K = 100, and a gain of 32% over the equilibrium surplus attained in that case. When marginal cost is equal to 4, the equilibrium and maximum surplus at the optimal capacity of 73.8 both fall to 743. This represents a gain of 19% over the maximum surplus at K = 100, and a gain of 93% over the equilibrium surplus in that case.

4 Simulations with Random Parameter Selection

In this section, we present a table showing the results of a set of simulations with random parameter selection for each of the outcomes discussed in sections 3 and 4. While supportive of the conclusions that we have already drawn in those sections, these results are important in that they provide a useful comparison of outcomes under both congestion etiquettes and competitive pricing (under the assumptions defined in Section 5). Since parameters for the computations below must be selected with some constraints in mind, the following results are not truly random, except within the specifications of those constraints. Nevertheless, these results may be considered to be more general than those previously reported, since they do not depend on the specific parameter sets chosen for individual examples.

Table 8 shows the results of the random simulations. Each entry in the table shows the relevant computation averaged over 100 randomly chosen parameter sets involving 6 (instead of 8) users. The random input values are chosen as follows: user values are random integers between 2 and 10; user demands are random integers between 1 and 25; user congestion factors are random numbers between 0.5 and 0.8; the capacity of Option B is equal to the total demand of all users multiplied by a random number between 0.3 and 1.0; and the price of Option A is a random number chosen between the minimum and maximum total surplus for each user.[26] Marginal cost in the competitive scenarios is

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[26] Recall from the discussion in Section 3 that undominated Nash equilibria under co-existence are guaranteed to achieve 100% total surplus efficiency when all users can afford Option A. Furthermore, expected total efficiency is necessarily small when no users can afford Option A. Actual efficiency in any particular random simulation depends on the specific parameters which define system capacity of Option B, the total demands of users, and their congestion limits.

assumed to be equal to 1. In cases where multiple equilibria exist, the reported result is an average of all equilibria.[27]

Table 8: Outcomes of Random Simulations

	Average TS	Average CS	TS Efficiency	CS Efficiency
Surplus Max (Original K_B)	379.87	251.31		
Co-existence	316.79	171.58	83.4%	68.3%
Pay to Connect	346.06	225.85	91.1%	89.9%
Simple Rand	289.25	195.08	76.1%	77.6%
Informed Rand	281.43	202.36	74.1%	80.5%
WTP	299.06	142.08	78.7%	56.5%
Competitive Nash Eq with Original K_B	343.63	180.95	90.6%	72.0%
Surplus Max (Optimal K_B)	425.82	287.81		
Competitive Nash Eq with Optimal K_B	363.33	212.99	85.3%	74.0%

The results shown in Table 8 may be briefly summarized as follows. When the capacity of Option B is fixed at its randomly chosen value as described above, the Pay to Connect Etiquette achieves superior outcomes to co-existence and all other etiquettes in terms of both total efficiency and consumer surplus efficiency. The remaining three congestion etiquettes (simple randomization, informed randomization and lump sum willingness to pay) do worse in terms of total and consumer surplus, but both of the randomization etiquettes achieve better outcomes than simple co-existence in terms of consumer surplus.

Results for the competitive outcomes, when capacity of Option B is set at its randomly chosen default level, show that in the competitive equilibrium, total efficiency

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[27] Note, however, that in the case of competitive equilibria with optimal capacity for Option B, the capacity is chosen in order to maximize total surplus, as explained in the previous section. Also, in this case, when there are multiple equilibria, Proposition 1 guarantees that at least one equilibrium achieves full efficiency in terms of total surplus.

is greater than the corresponding values both of the randomization etiquettes and for WTP. Total surplus efficiency is somewhat less than under the pay to connect etiquette. With optimally chosen capacity for Option B, both total surplus and consumer surplus are higher than each of the corresponding outcomes attained with the default capacity level. Since efficiency is in this case measured with respect to the maximum possible surplus (with optimal capacity instead of default capacity), the corresponding efficiencies reported are not, however, comparable to other values in the table.

5 Concluding Comments

The demand for wireless services can, at times, place a substantial burden on the transmission and, thus, the quality of service capabilities of the available spectrum. One approach to the resource congestion problem is to attempt to serve all users, thereby degrading service for all. Another response is to exclude some users in an effort to guarantee an acceptable quality of service for some. In this paper, we have examined the performance characteristics of a number of service etiquettes that might be utilized to reduce or eliminate congestion for users of unlicensed spectrum. We have demonstrated through examples that each of the etiquettes succeeds in this task, either by employing direct rationing of user demands or by using market pricing principles to allocate users to available spectrum. However, the same etiquettes that improve reliability in the congestible service option also have the effect of shifting demand for wireless use from traditional licensed use, with guaranteed service quality, to unlicensed use. Even when service etiquettes guarantee acceptable quality of service for some users, the net effect may be to lower total surplus.

Our analysis of each of the service etiquettes was done under the assumption that the total capacity of the congestible service was exogenously determined, and the prices (if any) charged to users of that service were not required to recover the cost of that service. In section 4 of the paper, we relaxed both of these assumptions. First, we incorporated a simple supply side to the model by imposing a zero profit constraint on the provider of the congestible service option, which we assumed to be supplied at constant marginal cost. If the capacity of the congestible service is fixed at the same level as assumed in the previous sections, we showed that when marginal cost is sufficiently small, the “competitive” model results in outcomes identical to those under the pay to connect etiquette. In many cases, both total surplus and consumers’ surplus are higher when consumers are required to bear the full cost of service for both licensed and unlicensed spectrum than in the model with free unlicensed capacity. When the capacity of the congestible service is allowed to vary, while the zero profit constraint is maintained, we were able to show that when capacity is chosen correctly, there exists a Nash equilibrium that maximizes total surplus, resulting in fully efficient user selections between the two service options.

Appendix A: Detailed Data from Examples

Table 9: Parameters for Example 1

Price of Option A: 130 Capacity of Option B: 100 Total Surplus Max: 770 Consumer Surplus Max: 528				
User (i)	Value per Unit (v_i)	Unit Demand (q_i)	Surplus (v_i q_i)	Congestion Limit
1	9	24	216	60
2	5	25	125	60
3	10	17	170	60
4	7	16	112	60
5	7	18	126	80
6	5	11	55	80
7	5	17	85	80
8	7	13	91	80

Table 10: Undominated Nash Equilibrium Assignments for Example 1 Assuming Co-Existence[28]

Option A	**Option B**	**Neither Option**	**Total surplus**	**Consumer surplus**
1, 3	2, 4, 5, 6, 7, 8		386	126

Table 11: Nash Equilibrium Assignment for Example 1 Assuming Pay to Connect Etiquette (Price of B = 1)

Option A	Option B	Neither Option	Total Surplus	Consumer Surplus
1, 3	5, 6, 7, 8	2, 4	743	479

Table 12: Nash Equilibrium Assignment for Example 1 Assuming the Simple Randomization Etiquette

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 3	2, 4, 5, 6, 7, 8		643.367	383.367

Table 13: Nash Equilibrium Assignment for Example 1 Assuming Congestion Informed Randomization Etiquette

Option A	Option B	Neither Option	Total Surplus	Consumer Surplus
1	2, 3, 4, 5, 6, 7, 8		560.25	430.25

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[28] There are 18 Nash equilibria altogether. Total surplus ranges from 386 to 743. Consumer surplus ranges from 126 to 483.

Table 14. Undominated Nash Equilibrium Assignments for Example 1 Assuming Lump-Sum Willingness to Pay Etiquette

Option A	Option B	Neither Option	Total surplus	Consumer surplus
	1, 2, 3, 4, 5, 6, 7, 8		512	137

Table 15: Parameters for Example 2

Price of Option A: 130		Capacity of Option B: 100		
Total Surplus Max: 1114		Consumer Surplus Max: 628		
User (i)	Value per Unit (v_i)	Unit Demand (q_i)	Surplus (v_i q_i)	Congestion Limit
1	8	19	152	60
2	10	17	170	60
3	6	16	96	60
4	10	22	220	60
5	7	13	91	80
6	8	18	144	80
7	9	19	171	80
8	5	14	70	80

Table 16: Undominated Nash Equilibrium Assignments for Example 2 Assuming Co-Existence[29]

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2, 4	3, 5, 6, 7, 8		1018	628

Table 17: Nash Equilibrium Assignment for Example 2 Assuming Pay to Connect Etiquette (Price of B = 1)

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2, 4	5, 6, 7, 8	3	1018	624

Table 18: Nash Equilibrium Assignment for Example 2 Assuming Simple Randomization Etiquette

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2, 4	3, 5, 6, 7, 8		1018	628

Table 19: Nash Equilibrium Assignment for Example 2 Assuming Informed Randomization Etiquette

Option A	Option B	Neither Option	Total surplus	Consumer surplus
4	1, 2, 3, 5, 6, 7, 8		609.593	479.593

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[29] There are two Nash equilibria in total. Both equilibria achieve the same total and consumer surplus values as reported in this table.

Table 20: Undominated Nash Equilibrium Assignments for Example 2 Assuming Lump-Sum Willingness to Pay Etiquette[30]

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2	3, 4, 5, 6, 7, 8		857	309
1, 4	2, 3, 5, 6, 7, 8		857	309
1, 6	2, 3, 4, 5, 7, 8		857	309
1, 7	2, 3, 4, 5, 6, 8		857	309
2, 4	1, 3, 5, 6, 7, 8		857	309
2, 6	1, 3, 4, 5, 7, 8		857	309
2, 7	1, 3, 4, 5, 6, 8		857	309
4, 6	1, 2, 3, 5, 7, 8		857	309
4, 7	1, 2, 3, 5, 6, 8		857	309
6, 7	1, 2, 3, 4, 5, 8		857	309

Table 21: Parameters for Example 3

Price of Option A: 50 Capacity of Option B: 100 Total Surplus Max: 1114 Consumer Surplus Max: 914				
User (i)	Value per Unit (v_i)	Unit Demand (q_i)	Surplus (v_i q_i)	Congestion Limit
1	8	19	152	60
2	10	17	170	60
3	6	16	96	60
4	10	22	220	60
5	7	13	91	80
6	8	18	144	80
7	9	19	171	80
8	5	14	70	80

Table 22: Undominated Nash Equilibrium Assignment for Example 3 Assuming Co-Existence[31]

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2, 3, 4	5, 6, 7, 8		1114	914

Table 23: Nash Equilibrium Assignment for Example 3 Assuming Pay to Connect Etiquette (Price of B = 1)

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2, 3, 4	5, 6, 7, 8		1114	910

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[30] There are 32 equilibria in all. Table 13 shows the 10 equilibria involving weakly un-dominated strategies. In the full set of Nash equilibria, total surplus is equal to 857 in each equilibrium, and consumer surplus ranges between 207 and 387.

[31] There are no additional equilibria involving weakly dominated strategies in this example.

Table 24: Nash Equilibrium Assignments for Example 3 Assuming Simple Randomization Etiquette

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2, 3, 4	5, 6, 7, 8		1114	914

Table 25: Nash Equilibrium Assignments for Example 3 Assuming Informed Randomization Etiquette

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2, 4, 6	3, 5, 7, 8		1007	807
1, 2, 4, 7	3, 5, 6, 8		1013.75	813.75
1, 4, 6, 7	2, 3, 5, 8		1114.0	914.0
2, 4, 6, 7	1, 3, 5, 8		1011.75	811.75

Table 26: Selected Nash Equilibria Assignments for Example 3 Assuming Willingness to Pay Etiquette[32]

Option A	Option B	Neither Option	Total surplus	Consumer surplus
1, 2, 3, 4, 5, 6, 7, 8			1114	714
1, 5, 6, 7,8	2, 3, 4		1114	714
4, 5, 6, 7, 8	1, 2, 3		1114	714
…	…	…	…	…

Table 27: Parameters for Example 4

Price of Option A: 250 Capacity of Option B: 100

Total Surplus Max: 561 Consumer Surplus Max: 561

User (i)	Value per Unit (v_i)	Unit Demand (q_i)	Surplus ($v_i q_i$)	Congestion Limit
1	8	19	152	60
2	10	17	170	60
3	6	16	96	60
4	10	22	220	60
5	7	13	91	80
6	8	18	144	80
7	9	19	171	80
8	5	14	70	80

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[32] There are 95 Nash equilibria involving undominated strategies. In each case total and consumer surplus are equal to 1114 and 714 respectively.

Table 28: Undominated Nash Equilibria Assignment for Example 4 Assuming Co-Existence[33]

Option A	Option B	Neither Option	Total surplus	Consumer surplus
	1, 2, 3, 4, 6, 7, 8		0	0

Table 29: Nash Equilibrium Assignment for Example 4 Assuming Pay to Connect Etiquette (Price of B = 1)

Option A	Option B	Neither Option	Total surplus	Consumer surplus
	5, 6, 7, 8	1, 2, 3, 4	476	472

Table 30: Nash Equilibrium Assignments for Example 4 Assuming Simple Randomization Etiquette

Option A	Option B	Neither Option	Total surplus	Consumer surplus
	1, 2, 3, 4, 5, 6, 7, 8		244.589	244.589

Table 31: Nash Equilibrium Assignments for Example 4 Assuming Informed Randomization Etiquette

Option A	Option B	Neither Option	Total surplus	Consumer surplus
	1, 2, 3, 4, 5, 6, 7, 8		420.975	420.975

Table 32: Undominated Nash Equilibrium Assignments for Example 4 Assuming Lump-Sum Willingness to Pay Etiquette[34]

Option A	Option B	Neither Option	Total surplus	Consumer surplus
	1, 2, 3, 4, 5, 6, 7, 8		561	105

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[33] There are 149 Nash equilibria altogether. Total and consumer surplus both lie between 0 and 476 in all equilibria.

[34] There are 32 equilibria in all. Table 13 shows the unique equilibrium involving weakly un-dominated strategies. In the full set of Nash equilibria, total surplus is equal to 561 in each equilibrium, and consumer surplus ranges between 105 and 351.